# Radiation Effect On Unsteady Mhd Convective Heat Mass Flow Of A Viscous Fluid Through A Porous Medium In A Vertical Channel With Oscillatory Wall Temperatures And Quadratic Density - Temperature Variation

[1]Dr. B. Suseelavathy
Department of Mathematics, S.S. & N College, Narasarao Pet, Guntur District, A.P., India
Email : lathabyneni@gmail.com

**ABSTRACT:**

We investigate the study make an attempt to analyse the effect of radiation unsteady convective heat transfer of a dissipative viscous fluid through a porous medium confined in a vertical channel on whose walls an oscillatory temperature is prescribed with quadratic density-temperature variation. Approximate solutions to coupled non-linear partial differential equations governing the flow and heat transfer are solved by a perturbation technique. The velocity, temperature,skin friction and rate of heat transfer are discussed for different variations of G, $D^{-1}$, $\alpha$, Ec, P, N and t.

**Keywords :** Viscous fluid, Porous medium, vertical channel, oscillatory wall temperature, quadratic density.

## 1. INTRODUCTION

Free convection flows between two long vertical plates have been studied for many years because of their engineering applications in the fields of nuclear reactors, heat exchangers, cooling appliances in electronic instruments. These flow were studied by assuming the plates at two different constant temperatures or temperature of the plates varying linearly along the plates etc. The study of fully developed free convection flow between two parallel plates at constant temperature was initiated by Ostrach (19). Combined natural and forced convection laminar flow with linear wall temperature profile was also studied by Ostrach (20).

Transient free convection flow between two long vertical parallel plates maintained at constant but unequal temperatures was studied by Singh *et al.*(28). Jha *et al.* (10) extended the problem to consider symmetric heating of the channel walls. Narahari *et al*. (15) analyzed the transient free convection flow between two long vertical parallel plates with constant heat flux at one boundary, the other being maintained at a constant temperature. Singh and Paul (28) presented and analysis of the transient free convective flow of a viscous incompressible fluid between two parallel vertical walls occurring as a result of asymmetric heating / coolong of the walls. Narahari (16)

In the context of space technology and in processes involving high temperatures, the effects of radiation are of vital importance. Recent developments in hypersonic flights, missile reentry, rocket combustion chambers, power plants for inter planetary flight and gas-cooled nuclear reactors have focused attention on thermal radiation as a mode of energy transfer and emphasize the need for improved understanding of radiative transfer in these processes. several authors have studies radiative heat transfer in chemical under vertical condision.

## 2. FORMULATION AND SOLUTION OF THE PROBLEM

We consider the non-Darcy unsteady flow of a viscous incompressible fluid through a porous medium in a vertical channel bounded by flat walls in the presence of constant heat sources. The unsteadiness in the flow is due to the oscillatory temperature prescribed on the boundaries. We choose a Cartesian coordinate system 0(x y) with walls at y = ± 1 by using Boussinesq approximation we consider the density variation only on the buoyancy term Also the kinematic viscosity υ,the thermal conductivity k are treated as constants. The equation governing the flow and heat transfer are

$$\frac{\partial u}{\partial t}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-(\frac{\mu}{k})u-(\frac{\sigma\mu_e^2 H_o^2}{\rho})u-\rho\,\overline{g} \tag{2.1}$$

$$\rho_0 C_p\frac{\partial T}{\partial t}=K_f\frac{\partial^2 T}{\partial y^2}+Q+2\mu(u_y^2\;)+\frac{\mu}{k}u^2-\frac{\partial(q_R)}{\partial y} \tag{2.2}$$

$$\rho - \rho_0 = -\beta_0(T - T_0)- \beta_1(T - T_0)^2 \tag{2.3}$$

where u is a velocity component in x-direction, T is a temperature, p is a pressure, ρ is a density ,k is the permeability of the porous medium,μ is dynamic viscosity, $k_f$ is coefficient of thermal conductivity, β is coefficient of volume expansion and Q is the strength of heat source

The boundary conditions are

$$\left.\begin{aligned} &u = 0,\quad T = T_1 \text{ at } y = -L\\ &u = 0,\; T = T_1 + \in(T_2 - T_1)\cos\omega t \end{aligned}\right\} \tag{2.4}$$

on introducing the non dimensional variables

$$y' = y/L,\qquad u'=\frac{u}{(\nu/L)}\qquad \theta=\frac{T-T_1}{T_2-T_1},\qquad t' = \omega t,$$

Equations 2.1 & 2.2 reduce to (dropping the dashes)

$$\gamma_1^2\frac{\partial u}{\partial t}=G(\theta+\gamma\theta^2)+\frac{\partial^2 u}{\partial y^2}-(D^{-1}+M^2)u \tag{2.5}$$

$$P\gamma_1^2\frac{\partial\theta}{\partial t}=(1+\frac{4}{3N})\frac{\partial^2\theta}{\partial y^2}+\alpha+PE_c u_y^2+P\,E_c D^{-1}u^2 \tag{2.6}$$

where

$$G=\beta g L^3\frac{(T_2-T_1)}{\gamma^2}\qquad \text{(Grashof number)}$$

$$D^{-1}=\frac{L^2}{k}\qquad \text{(Darcy parameter)}$$

$$P=\frac{\mu C_P}{K_f}\qquad \text{(Prandtl number)}$$

$$\alpha=\frac{Q.L^2}{(T_1-T_2)K_f}\qquad \text{(Heat source parameter)}$$

$$Ec=\frac{\mu^2}{C_p L^2(T_2-T_1)}\qquad \text{(Eckert Number)}$$

$$N=\frac{4\sigma^* T_e^3}{3\beta_R}\qquad \text{(Radiation parameter)}$$

$$\gamma_1^2=\frac{\omega L^2}{\nu}\qquad \text{(Wormsely Number)}$$

$$\gamma^2 = \frac{\beta_1(T_2 - T_1)}{\beta_0} \qquad \text{(Density ratio)}$$

$$P_1 = \frac{3NP}{3N+4} \quad \alpha_1 = \frac{3N\alpha}{3N+4}$$

$$M_1^2 = M^2 + D^{-1}$$

The transformed boundary conditions are

$$\left.\begin{array}{ll} u = 0, \quad \theta = 0, & \text{at } y = -1 \\ u = 0, \quad \theta = 1 + \in \cos(\omega t) & \text{at } y = +1 \end{array}\right\} \qquad (2.7)$$

**3.METHOD OF SOLUTION**

In view of the boundary condtions (2.4) we assume

$$u = u_0 + \in e^{it} u_1 \qquad (2.8)$$

$$\theta = \theta_0 + \in e^{it} \theta_1$$

Substituting the series expansion (2.8) in equations( 2.5)&(2.6) and separating the steady and transient terms we get

$$\frac{\partial^2 u_0}{\partial y^2} - M_1^2 u_0 = -G(\theta_0 + \gamma\theta_o^2) \qquad (2.9)$$

$$\frac{\partial^2 u_1}{\partial y^2} - (M_1^2 + i\gamma^2)u_1 = -G(\theta_1 + 2\theta_0\theta_1) \qquad (2.10)$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + \alpha_1 + P_1 E_c \frac{\partial^2 u_0}{\partial y^2} + P_1 E_c D^{-1} u_0^2 = 0 \qquad (2.11)$$

$$\frac{\partial^2 \theta_1}{\partial y^2} - (iP\gamma^2)\theta_1 + (2P_1 Ec)\frac{\partial u_0}{\partial y}.\frac{\partial u_1}{\partial y} + (P_1 EcD^{-1})u_0 u_1 \qquad (2.12)$$

Since the equations (2.9 – 2.12) are non-linear coupled equations., assuming Ec << 1 we take

$$u_0 = u_{00} + Ec\, u_{01}$$

$$u_1 = u_{10} + Ec\, u_{11}$$

$$\theta_0 = \theta_{00} + Ec\, \theta_{01} \qquad (2.13)$$

$$\theta_1 = \theta_{10} + Ec\, \theta_{11}$$

Substitutimg (2.13) in equations (2.9) – (2.12) and separating the like terms we get

$$u_{00}^{11} - M_1^2 u_{00} = -G(\theta_{00} + \gamma\theta_{00}\theta_{01}), \quad u_{00}(\pm 1) = 0 \qquad (2.14)$$

$$\theta_{00}^{11} = -\alpha_1, \quad \theta_{00}(-1) = 0,\ \theta_{00}(+1) = 1 \qquad (2.15)$$

$$u_{01}^{11} - M_1^2 u_{01} = -G(\theta_{01} + 2\theta_{00}\theta_{01}), u_{01}(\pm 1) = 0 \qquad (2.16)$$

$$\theta_{01}^{11} = -P_1 u_{00}^{12} - P_1 D^{-1} u_{00}^2, \quad \theta_{01}(-1) = 0 = \theta_{01} \qquad (2.17)$$

$$u_{10}^{11} - (M_1^2 + i\gamma^2)u_{10} = -G(\theta_{10} + 2\gamma\theta_{00}\theta_{10}), u_{10}(\pm 1) = 0 \qquad (2.18)$$

$$\theta_{10}^{11} - iP_1\gamma_1^2\theta_{10} = 0, \quad \theta_{10}(-1) = 0, \quad \theta_{10}(+1) = 1 \qquad (2.19)$$

$$u_{11}^{11} - (M_1^2 + i\gamma_1^2)u_{11} = -G\theta_{11}, \ u_{11}(\pm 1) = 0 \qquad (2.20)$$

$$\theta_{11}^{11} - (iP_1\gamma_1^2)\theta_{11} = -2P_1 u_{00}^1 u_{10}^1 - 2P_1 D^{-1} u_{00} u_{10}, \theta_{11}(\pm 1) = 0 \qquad (2.21)$$

Solving the equations (2.14)-(2.21) subject to the relevant boundary conditions we obtain

$$\theta_{oo} = \frac{\alpha_1}{2}(1 - y^2) + 0.5(y+1)$$

$$u_{00} = a_8(1 - \frac{Ch(M_1 y)}{Ch(M_1)}) + a_{10}(y^2 - \frac{Ch(M_1 y)}{Ch(M_1)}) - a_{12}(y^4 - \frac{Ch(M_1 y)}{Ch(M_1)}) + + a_9(y - \frac{Sh(M_1 y)}{Sh(M_1)}) - a_{11}(y^3 - \frac{Sh(M_1 y)}{Sh(M_1)})$$

$$\theta_{01} = a_{51}y^2 + a_{52}y^3 + a_{53}y^4 + a_{54}y^5 + a_{55}y^6 + a_{56}y^7 + a_{57}y^8 + + a_{58}y^9 + a_{59}y^{10} + a_{60}Ch(2M_1 y) + a_{61}Sh(2M_1 y) + + (a_{62} + ya_{64} + y^2 a_{66} + y^3 a_{68} + y^4 a_{70})Sh(M_1 y) + + (a_{63} + ya_{65} + y^2 a_{67} + y^3 a_{69} + y^4 a_{71})Ch(M_1 y) + + a_{72}y + a_{73}$$

$$u_{01} = b_1(1 - \frac{Ch(M_1 y)}{Ch(M_1)}) + b_2(y - \frac{Sh(M_1 y)}{Sh(M_1)}) + b_3((y^2 - \frac{Ch(M_1 y)}{Ch(M_1)}) + + b_4((y^3 - \frac{Sh(M_1 y)}{Sh(M_1)}) + b_5(y^4 - \frac{Ch(M_1 y)}{Ch(M_1)}) + b_6(y^5 - \frac{Sh(M_1 y)}{Sh(M_1)}) + + b_7(y^6 - \frac{Ch(M_1 y)}{Ch(M_1)}) + b_8((y^7 - \frac{Sh(M_1 y)}{Sh(M_1)}) + b_9(y^8 - \frac{Ch(M_1 y)}{Ch(M_1)}) +$$

$$+ b_{10}(y^9 - \frac{Sh(M_1 y)}{Sh(M_1)}) + b_{11}(y^{10} - \frac{Ch(M_1 y)}{Ch(M_1)}) + b_{12}(y^{11} - \frac{Sh(M_1 y)}{Sh(M_1)}) + + b_{13}((y^{12} - \frac{Ch(M_1 y)}{Ch(M_1)}) + b_{14}(Ch(2M_1 y) - Ch(2M_1)) + b_{15}(Sh(2M_1 y) - - ySh(2M_1)) + (b_{16} + yb_{18})(ySh(M_1 y) - Sh(M_1)) + b_{17}y(Ch(M_1 y) - - Ch(M_1)) + (b_{19} + yb_{21})(y^2 Ch(M_1 y) - Ch(M_1)) + b_{20}(y^3 Sh(M_1 y - - Sh(M_1) + b_{22}y(y^3 Sh(M_1 y) - Sh(M_1)) + b_{23}(y^4 Ch(M_1 y) - Ch(M_1)) + + b_{24}(y^5 Sh(M_1 y) - Sh(M_1)) + b_{25}y(y^4 Ch(M_1 y) - Ch(M_1)) +$$

$$+ b_{26}y(y^5 Sh(M_1 y) - Sh(M_1))$$

$$\theta_{10} = 0.5(\frac{Ch(\beta_2 y)}{Ch(\beta_2)} + \frac{Sh(\beta_2 y)}{Sh(\beta_2)})$$

$$u_{10} = b_{45} Ch(\beta_3 y) + b_{46} Sh(\beta_3 y) + \phi_4 (y)$$

$$\phi_4 (y) = (b_{39} + y b_{42} + y^2 b_{44}) Sh(\beta_2 y) + (b_{40} + y b_{41} + y^2 b_{43}) Ch(\beta_2 y)$$

where $a_1, a_2, \ldots\ldots\ldots a_{73}, b_1, \ldots\ldots\ldots b_{43}$ are constants given in the Appendix.

## 4. REFERENCES


1) Abd El-Naby M.A, Elsayed M.E., Elberabary and Nader Y.A. (2004). Finite difference solution of radiation effects on MHD free convection flow over a vertical porous plate, Appl. Maths Comp. Vol. 151, pp. 327-346.

2) Anug W (1972). Fully developed laminar free convection between vertical plates heated asymmetrically. *Int. J. Heat and Mass Transfer*, 15, pp.1577-1580.

3) Bakier A.Y. and Gorla R.S.R. (1996). Thermal radiation effects on mixed convection from horizontal surfaces in porous media, Transport in porous media, Vo. 23 pp. 357-362.

3a) Brinkman,H,C(1948)A calculation of the viscous force external by a flowing fluid on a dense swam of particles:Application Science Research.Ala.P.81

4) Chamkha A.J., Takhar H.S. and Soundalgekar V.M(2001) Radiation effects on free convection flow past a semi-infinite vertical plate with mass transfer, Chem. Engg. J, Vol. 84, pp. 335-342.

5) Chamkha A.J. (1997) Solar Radiation Assisted natural convection in a uniform porous medium supported by a vertical heat plate, ASME Journal of heat transfer, V.19, pp.89-96.

6) Chitrphiromsri, P and Kuznetsov, A.V. (2005). Porous medium model for investigating transient heat and moisture transport in firefighter protective clothing under high intensity thermal exposure, J. Porous media, Vo. 8,5, pp. 10-26.

7) Campo A, Manca O, and Marrone B (2006). Numerical investigation of the natural convection flows for low-Prandtl fluid in vertical parallel-plates channels. *ASME Journal of Applied Mechanics,* 73, pp. 6-107.