TERNARY Commutative Semigroups

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ABSTRACT:-In this paper mainly we have obtained certain properties of Ternary semi groups and commutative ternary semigroups and sub ternary semigroups.

INTRODUCTION:-The concept of ternary semigroups were introduced by LEHMER. Previously the structure of n-ary algebras were studied by KASNER ;Ternary semi groups are universal algebra with a single operation satisfies associative law ;In this paper mainly we have obtained certain examples of a ternary semi groups and certain theorems basing on commutative ternary semigroups and sub ternary semigroups. A ternary semigroup T is a non-empty set in which there exists a function from TxTxT to T which satisfies the condition

$[(x_1x_2x_3)x_4x_5] \quad =[x_1(x_2x_3x_4)x_5] = [x_1x_2(x_3x_4x_5)] \ \forall\ x_i \epsilon T, \ 1 \le i \le 5;$

It is easily observed that a ternary semi group T need not be a semi group and any semi group can be reduced to a ternary semi group; A non empty sub set M of a Ternary semi group T is called a sub ternary semi group .

If for any a,b,c ϵM=>abc ϵM

It Is observed in this paper that the intersection of any two sub ternary semigroup is also a sub ternary semi group and also the arbitrary family of any sub ternary semigroups of T is also sub ternary semi group . It is observed in this paper that the union of any two sub ternary semi groups need not be a sub ternary semigroup. Commutative ternary semi groups and quasi commutative semi groups are introduced. We obtained a result that any commutative ternary semi group is quasi commutative but the converse need not be true.

First we start with the following preliminaries.

Def 1: A non empty set together with a binary operation satisfying associative law is called a semigroup

Following are certain examples of semi group.

Ex 1:1) The set of natural numbers under usual addition and usual multiplication is a semi group .

2) The set { 1,-1} is a semi group under usual multiplication.

Now we have introduced Ternary semi group.

Def 2: A non empty set T is said to be a ternary semi group if there exists a mapping TXTXT->T which maps (x_1,x_2,x_3)---->$[x_1x_2x_3]$ satisfy the condition

$[(x_1x_2x_3)x_4x_5] = [x_1(x_2x_3x_4)x_5]$

Remark 1: Any ternary semi group need not be a semi group.

Remark 2: If A,B,C are any three subsets of T then ABC ={abc: aϵA, bϵB, cϵC}

From the following example it is observed that any ternary semigroup need not be a semi group .

Ex 1: Define T={i,-i} is a ternary semi group under multiplication of complex numbers but it is not a semi group.

Remark 3: Any semi group can be extended to a ternary semi group .

The following examples of ternary semi groups.

Ex 2: Let T={0,a,b} and *is an operation defined on T by

(x*y)*z=xyz ∀ x,y,zϵT whose composition table is

*	0	a	b
0	0	0	0
a	0	a	a
b	0	b	b

Ex 3: let T={0,1,2,3,4,5} and define *on T by (a*b)*c=abc ∀ a,b,c ϵ T

Whose composition table is

*	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	1	1	1	1
2	0	1	2	3	1	1
3	0	1	1	1	2	3
4	0	1	4	5	1	1
5	0	1	1	1	4	5

Ex 4: Let $T=z^{+}*z^{+}$, Define * on T by (a,b)*(c,d)*(e,f)=(a,f)

Then T is a ternary semi group sub ternary semi group is defined as follows .

Def 3: A non empty subset M of T is called a ternary sub semi group

if for any a,b,c ϵ M => abc ϵ s.

Remark 4: A non empty subset M of a ternary semi group T is a ternary sub semigroup

if MMM $\subseteq$ T.

The following theorem shows that the intersection of any two ternary sub semigroups of T is also a ternary sub semi group.

Theorem 1: Intersection of any two ternary sub semi groups of T is also a ternary sub semi group.

Proof : Let M_1,M_2 be any two ternary sub semi groups of a ternary semi group T then $M_1 \cap M_2$ is also a ternary sub semi group let x,y,z ϵ $M_1 \cap M_2$

Imply that x,y,z ϵ M_1 and x,y,z ϵ M_2.

since M_1 is a ternary sub semi group of T

x,y,z ϵ M_1 =>x.y.z ϵ M_1

And since M_2 is a ternary sub semi group of T

=>x,y,z ϵ M_2

Hence xyz ϵ $M_1 \cap M_2$ for any x,y,z ϵ $M_1 \cap M_2$

Hence $M_1 \cap M_2$ is also a ternary sub semi group of T.

The following theorem shows that the intersection of an arbitrary ternary sub semi groups of T is also a ternary sub semi group.

Theorem 2: The intersection of arbitrary family of ternary sub semigroups of T is also a ternary sub semi group .

Proof: let $\{M_{\dot\alpha}\}_{\dot\alpha \in \Delta}$ be an arbitrary family of ternary sub semi groups with x,y,z ϵ $\cap_{\dot\alpha \in \Delta} M_{\dot\alpha}$

Imply that x,y,z ϵ $M_{\dot\alpha}$

As each $M_{\dot{\alpha}}$ is a ternary sub semi group

=> xyz ϵ $M_{\dot{\alpha}}$ ∀ $\dot{\alpha}$

=> xyz ϵ $\cap_{\dot{\alpha}\epsilon\Delta}$ $M_{\dot{\alpha}}$

Hence $\{M_{\dot{\alpha}}\}_{\dot{\alpha}\epsilon\Delta}$ is also a ternary sub semi group .

Remark 5: It is easy to observe that the union of any two ternary subsemi groups of T, need not be a ternary sub semi group.

Now Commutative ternary semi groups are defined as follows .

Def 3: A ternary semi group T is said to be commutative if for any a,b,c ϵ T, abc=bca=cab=bac=cba=acb.

Remark 6: It is easy to observe that a commutative semi group is a commutative ternary semi group where as any commutative ternary semi group need not be commutative in semi group.

Quasi commutative ternary semigroups are defined as follows.

Def 4: A ternary semi group T is said to be quasi commutative if for any a,b,c ϵ T,there exists

n ϵ N such that

$abc=b^nac=bca=c^nba=cab=a^ncb$

From the following theorem it is observed that a commutative ternary semi group is quasi commutative

Theorem 3: If T is a commutative ternary semi group then T is quasi commutative

Proof : let T be a commutative ternary semi group then for any a,b,c ϵ T imply that

abc=bca=cab=bac=cba=acb

$\Rightarrow abc=b^1ca=bca=c^1ba=cab=a^1cb$

imply that T is quasi commutative

Remark 7: The converse of the above theorem need not be true.

Normal ternary semigroups are defined as follows.

Def 5: A ternary semi group T is said to be normal if abT =Tab , ∀ a,b ϵ T

From the following theorem it is observed that any quasi commutative semi group is a normal ternary semi group .

Theorem 4: If T is a quasi commutative ternary semi group then T is a normal ternary semi group.

Proof: let T be a quasi commutative and let a,b є T with x є abT

Then x= abc for some c є T

Since T is quasi commutative =>x=abc=c^nab єTab

Hence abT ⊆ Tab ------→*

Conversely let xєTab=>x=cab for some cєT

As T is quasi commutative =>x=cab=abcєabT

Hence Tab ⊆ abT-----→**

From *and ** Tab =abT so that the ternary semi group T is normal.

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