Optimal approach for Transportation Problems of four Variables

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Abstract: *Transportation problem is considered a enormously important aspect that has been studied in a wide range of operations including research domains. As such, it has been used in simulation of several real life problems. Thus, optimizing transportation problem of variables has remarkably been significant to various disciplines. In this paper, four variables will be optimized to reduce transportation cost using four methods which will include: Northwest corner cell method, least cost method, Vogel's approximation method and modi(modified distribution) method. This will mainly aim at finding the best and cheapest route on how supply will be used to satisfy demand at specific points. This will imply that, a variable cost of shipping products from suppliers to demand points will be considered.*

Key words: *optimization techniques, transportation problem, Northwest corner, least cost, Vogel, Modi.(Modified distribution.)*

Optimization refers to the process of choosing elements considered to be the best from several alternatives that might be availed. As such, one has to solve problems with the aim of minimizing or maximizing a real function. This can be achieved via choosing values of integers or real variables from a specified set of values. This makes transportation to be considered as simply being after finding an optimal distribution plan for a certain single commodity (Ahmad 2012). When commodity supply is available at various sources, demand tends to be specified for the commodity at every destination, with transportation cost from source to destination clearly defined. In this case, the puzzle is in finding the optimal distribution plan that can minimize the overall transportation cost for product transportation from sources to destinations.

In this paper, transportation problem will be formulated as linear programming problems that will be solved using four methods1 (Atoum 2009). The MODI method is considered as being a standardized technique when it comes to obtaining optimal solution. On the other hand, Vogel is believed to be an approximation method. It usually tends to produce an optimal or near optimal initial solution. Several researches in this field determined that Vogel produces an optimum solution in almost 85% of the problems under test. Advantage of this method is that it accounts for its allocations.

In addition to the above two methods, the least cost method achieves its goals via giving more allocations to the least cost cell. Rows and columns with complete allocations are ignored as the allocation process continues (Soewono 2010). This procedure is considered to be complete on condition that all requirements for all rows and columns have satisfactorily been addressed. The Northwest corner is the forth method and it begins its allocation at the northwest corner of the matrix. It ensures that it assigns more units to each cell while putting into consideration to meet the requirement of not having more that $m+n-1$ filled cells. In this case, m=number of rows while n=number of columns . This procedure is iterated for the remaining rows until when the requirements for all rows and columns will have been met.

II. TRANSPORTATION METHOD

When transportation method is employed in solving a transportation problem, the very initial step that has to be undertaken is to obtain a feasible solution satisfying demand and supply requirement values. Three methods will be used in this paper to obtain this initial basic solution. As mentioned earlier in this paper, the methods to be used will include:

2.1 Algorithm for northwest- corner cell method

This is a method used to compute feasible solution of a transportation problem. In this method, the basic variables are usually chosen from the top left corner commonly referred to as the Northwest corner cell. The following steps are followed to obtain this feasible solution

2.1.1 Find the minimum of the supply and demand values with respect to the current northwest-corner cell of the cost matrix.
2.1.2 Allocate this minimum value to the current northwest-corner cell and subtract this minimum from the supply and demand values with respect to the current northwest-corner cell.
2.1.3 Check whether exactly one of the row/column corresponding to the northwest-corner has zero supply/demand, respectively. If yes go to step 2.1.4 otherwise go to step 2.1.5.
2.1.4 Delete that row/column with respect to the current northwest corner cell which has the zero supply/demand and go to step 2.1.6.
2.1.5 Delete both row and column with respect to the current northwest-corner cell.
2.1.6 Check whether exactly one row or column is left out. If yes, go to step 2.1.7 otherwise go to step 2.1.1.
2.1.7 Match the supply/demand of that row/column with the remaining demands/supplies of the undeleted columns/rows.
2.1.8 Go to phase 2.

Example: Consider the problem of allocating raw materials from four different warehouses to four different plants. The availability of the raw material in the four warehouses are 14units, 10units, 15units, and 12units.The demand of the raw material in the four plants are10 units, 15 units, 11 units, and 15 units. The following table 1 gives a summary of shipping cost per unit.

10	30	25	15
20	15	20	10

10	30	20	20	**warehouse**
30	40	35	45	

Plant

Use the following methods to obtain a feasible solution in order to minimize their total transportation cost: Northwest corner, least cost method, Vogel's approximation method and MODI method.

Mathematical statement of the above transportation problem:

Mathematically, transportation can be defined as:

Minimize $Z=\sum_{i=1}^{m} c_{ij} \sum_{j=1}^{n} X_{ij}$

Destination(j)

1 2 ……..j ……n

1

2

i

:

j

Source(i) :

$$\begin{matrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2j} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ c_{i1} & c_{i2} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mj} & \cdots & c_{mn} \end{matrix}$$

$\sum_{j=1}^{n} X_{ij} \le a_i$, i=1,2,…..,m. and

$\sum_{i=1}^{m} X_{ij} \ge b_j$, j=1,2,……n. where $X_{ij} \ge 0$,i=1,2,……,m. and j=1,2,…..,n.

The objective function minimizes the total cost of transportation (Z) between various sources and destinations. The constraints i in the first set of constraints ensures that the total units transported from source I is less than or equal to its supply. The constraint j in the second set of constraints ensures that the total units transported to the j is greater than or equal to its demand.

c_q = quantity produced at the plants, q= plants, d_p = quantity required at the warehouse, p = warehouses, e_{pq} = cost of transportation for a single unit from plants to warehouses and f_{dp} = transported quantity from plants to warehouses.

Thus, using the Northwest-corner cell method, the basic feasible solution can be obtained as shown in table 2 below:

Table2

Source(i)	Destination(j) 1	2	3	4	supply
1	10[10]	30[4]	25	15	14
2	20	15[10]	20[3]	10	10
3	10	30[1]	20[11]	20	15
4	30	40	35	45[12]	12
Demand	10	15	11	15	**51**

The total cost in this method will be obtained as follows:

Total cost= (10*10)+(30*4)+(15*10)+(20*3)+(30*1)+(20*11)+(45*12) =1220 units

As per the rule of this method, this unit routing has managed to satisfy all the demand and supply requirements via involving. 7(= m+n-1= 4+4-1).

2.2Algorithm for least cost method :

When least cost as a method is used to compute a basic feasible solution in a transportation problem, choosing basic variables is usually done as the unit cost of transportation. To obtain this feasible solution, the following steps have to be considered and followed:

2.2.1 Find the minimum of the(undeleted)values in the cost matrix(i.e find the matrix minimum).

2.2.2 Find the minimum of the supply and demand values(X)with respect to the cell corresponding to the matrix minimum.

2.2.3 Allocate X units to the cell with the matrix minimum. Also, subtract X units from the supply and demand values corresponding to the cell with the matrix minimum.

2.2.4 Check whether exactly one of the row/column corresponding to the cell with the matrix minimum has zero supply/zero demand, respectively. If yes go to step 2.2.5 otherwise go to step 2.2.6.

2.2.5 Delete that row/column with respect to the cell with the matrix minimum which has the zero supply/zero demand and go to step 2.2.7.

2.2.6 Delete the both row/column with respect to the cell with the matrix minimum.

2..2.7 Check whether exactly one row/column is left out. If yes go to step 2.2.8 otherwise, go to step 2.2.1.

2.2.8 Match the supply/demand of that row/column with the remaining demands/supplies of the undeleted columns/rows.

2.2.9 Go to phase 2

Table3

Destination(j)

Source(i)		1	2	3	4	supply
	1	10[10]	30	25	15	14
	2	20	15[4]	20	10[10]	10
	3	10	30	20[11]	20[1]	15
	4	30[3]	40[12]	35	45	12
Demand		10	15	11	15	**51**

The total cost in this method will be obtained as follows:
Total cost=(10*10)+(10*10)+(15*4)+(20*11)+(20*1)+(30*3)+(40*12)=1070 units.

Just like in the Northwest corner method, the least cost method has also managed to satisfy demand/supply requirement as it involved. 7(= m+n-1 = 4+4-1).

2.3Algorith for Vogel's approximation method:

This method tends to repeat procedures to compute a basic feasible solution of the transportation problem. However, some steps have to be followed while carrying out such computations. These include:

2.3.1 Find the row penalties, i.e. the difference between the first minimum and the second minimum in each row. If the two minimum values are equal, then the row penalty is zero.
2.3.2 Find column penalties, i.e. the difference between the first minimum and second minimum in each column. If the two minimum values are equal, then the column penalty is zero.
2.2.3 Find the maximum amongst the row penalties and column penalties and identify whether it is occur in a row /column(break tie randomly). If the maximum penalty is in a row go to step 2.3.4otherwise, go to step 2.3.7.
2.3.4 Identity the cell for the allocation which has the least cost in that row.
2.3.5 Find the minimum of the supply and demand values with respect to the selected cell values.
2.3.6 Allocate this minimum value to the cell and subtract this minimum from the supply and demand values with respect to the selected cell and go to step 2.3.10.
2.3.7 Identify the cell for allocation which has the least cost in that column.
2.3.8 Find the minimum of the supply and demand values with respect to the selected cell values.
2.3.9 Allocate this minimum value to the cell and subtract this minimum from the supply and demand values with respect to the selected cell.
2.3.10 Check whether exactly one of the rows and the columns corresponding to the selected cell has zero supply/zero demand, respectively. If yes go to step 2. 3.11; otherwise go to step 2.3.12.
2.3.11 Delete the row or column which has the zero supply/zero demand and revise the corresponding row/column penalties. Then go to step 2.3.13.
2.3.12 Delete the both row and column with respect to the selected cell. Then revise the row and the column penalties.
2.3.13 Check whether exactly one row/column is left out. If yes, go to step 2.3.14,otherwise, go to step2.3.3.
2.3.14 Match the supply/demand of the left-out row/column with the remaining demands and supplies of the undeleted columns/rows.
2.3.15 Go to phase 2.

Table4

Destination(j)

1	2	3	4	supply	rowpenalty
10	30	25	15[14]	14	5

1	20	15[10]	20	10	10	5
Source(i) 2	10[10]	30	20[4]	20[1]	15	10
	30	40[5]	35[7]	45	12	5
3 4	10	15	11	15	**51**	
Demand c.penalty	0	15*	0	5		

The total cost in this method will be obtained as follows:
Total cost=(15*14)+(10*15)+(10*10)+(20*4)+(20*1)+(40*5)+(35*7)=1005 units.

Just like with the previous methods, Vogel method also managed to satisfy supply and demand requirements.
After obtaining initial solution using these first three methods, it has to pass through an optimal testing process. By definition, an optimal solution refers to a solution in which no more transportation routes can manage to reduce the total transportation cost (Chaudhuri 2011). This implies that the process of evaluating cells that are not occupied in a transportation table has to be initiated. This process is usually carried out in terms of opportunities aiming at reducing the overall transportation cost (Araghi 2009). The new set of transportation routes must then include a cell having the largest negative opportunity cost but remain unoccupied. In such cases, the forth method mentioned is considered to be the most efficient. This method is called the MODI method.

2.4 Modified distribution method(MODI)
To obtain this optimal solution using this method, a number of steps have to be followed. These include:
Step1: Row1,row 2,…..row m of the cost matrix are assigned with variables $U_{1,}$, U_2,…. U_m ,respectively and the column 1,column2,……column n are assigned with variables V_1, V_2,…. V_n respectively.
Step2: Check whetherthe number of basic cells in the set of initial basic feasible solution is equal to m+n-1.If yes, go to step 4, otherwise, go to step 3.

Step3: Convert the necessary number of non-basic cells into basic cells to satisfy the condition stated step2.The concept of closed loop explained in step8.

Step4: Compute the values for $U_{1,}$, U_2,.... U_m, and V_1, V_2,.... V_n by applying the following formula to all the basic cells only.

$U_i + V_j = C_{ij}$ (assume $U_{1,}$,=0)

Step5: Compute penalties P_{ij} for the non-basic cells by using the formula:

$P_{ij} = U_i + V_j - C_{ij}$

Step6: Check whether all P_{ij} values are less than or equal to zero. If yes , go to step12,otherwise,go to step 7.

Step7: Identify the non-basic cells which has the maximum positive penalty, and term that cell as the new basic cell.

Step8: Starting from the new cell, draw a closed loop consisting of only horizontal and vertical lines passing through some basic cells.

Step9: Starting from the new basic cell, alternatively assign positive(+) and negative(-) signs at the corners of the closed loop.

Step10: Find the minimum of the allocations made amongst the negatively signed cells.

Step11: Obtain the table for the next iteration by doing the following steps and then go to step2.

(i) Add the minimum allocation obtained the previous step to all the positively signed cells and subtract minimum allocation from all the negatively signed cells and then treat the net allocations as the allocations in the corresponding cells of the next iteration.

(ii) Copy the allocations which are on the closed loop but not at the corner points of the closed loops, as well as the allocations which are not on the closed loop as such without any modifications to the corresponding cells of the next iteration.

Step12: The optimality is reached. Treat all present allocations to the set of basic cells as the optimum allocations.

Step13: Stop.

Table5

Source(i) \ Destination(j)	1	2	3	4	supply	U_i values
1	10	30	25	15[14]	14	U_1=0
2	20	15[10]	20	10	10	U_2=-5
3	10[10]	30	20[4]	20[1]	15	U_3=5
4	30	40[5]	35[7]	45	12	U_4=20
Demand	10	15	11	15	**51**	
V_j values	V_1=5	V_2=20	V_3=15	V_4=15		

The total cost in this method will be obtained as follows:
Total cost=(15*14)+(10*15)+(10*10)+(20*4)+(20*1)+(40*5)+(35*7)=1005 units.

III. Conclusion

This paper considered optimization techniques of transportation problem for four variables using four methods. Optimization of problems is the same as choosing optimal solution from the available alternatives. As evidenced from previous researches, this technique is applicable in a wide range of fields. However, MODI method employed in this paper can be used with good deal of success in solving such problems. This is because while working hand in hand with the remaining three methods, MODI method computed the optimal solution within a shorter period of time. Besides, it reduced complexities via producing a simple and clear solution that could be easily used in other areas for optimizing other problems.

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