Mathematical Modeling in other Disciplines

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ABSTRACT

The impact of modern mathematics and its applications in other disciplines is presented from the 20th century historical perspective. In the period 1930's to 1970's mathematics became more inward looking and the distinct0ion between pure and applied mathematics became much more pronounced. In 1970's, there was a return to more classical topics but on a new level and this resulted in a new convergence between mathematics and physics. The 20th century approach to mathematics resulted in a more developed mathematical language, new powerful mathematical tools and inspired new application areas that have resulted in tremendous discoveries in other applied sciences. Towards the end of the 20th century, mathematicians were making a re-think on the need to bridge the division lines within mathematicians, to open up more for other disciplines and to foster the line of inter-discipline research. The current century cry is that this interaction will be further strengthened in the 21st century.

INTRODUCTION

Mathematics has been vital to the development of civilization. From ancient to modern times, mathematics has been fundamental to advances in science, engineering and philisophy. Developments in modern mathematics have been driven by a number of motivations that can be categorized into the solution of a difficult problem and the creation of new theory enlarging the fields of applications of mathematics. Very often the solution of a concrete difficult problem is based on the creation of a new mathematical theory. While on the other hand creation of a new mathematical theory may lead to the solution of an old classical problem. This paper is discussing the current role of mathematics in other disciplines.

Trend s of Applications in the 21st century:

The 21st century made a re-think on the foundations of mathematics. It was characterized by a new approach to mathematics, fuelled by David Hilberts (1862-1943) famous set of "Mathematical problems in the 1990 International Congress of Mathematicians. Hilbert's vision was to analyse axioms of each subject and state results in their full generality. This vision became concrete in the 1930's through the development of the axiomatic approach to algebra, pioneered by E.Artin and Edith Noether, parallel trends took place in functional analysis with Bianach Spaces. This spread rapidly to algebraic topology, harmonic analysis and partial differential equations. In addition to this axiomatic approach, the Bourbaki group introduced the idea that there was one universal set of definitions, which once learnt would be the foundations of everything more specialized (Mumford, 1998). In the drive to seek generality, 20th century mathematics became more diverse, more structured and more complex.

Divergence of Mathematics from Physics:

In the 18th and 19th century mathematical language was vague and did not allow much interaction among mathematicians of different fields. In the period 1950's to 1970's Mathematicians concentrated around problems of algebraic topology, algebraic geometry and complex analysis and they developed new concepts and new methods. New powerful mathematical tools were developed and the language of mathematics became highly developed and very powerful. This has had great impact on diverse fields such as number theory, set theory, geometry, topology and partial differential equations. This new approach to mathematics resulted in greater abstraction. Mathematicians spent years of apprenticeship in a full set of abstraction before doing their own thinking. When the basis were clear enough there was a search for powerful tools that allowed for development and expansion of the geometric intuition into new domains. Examples are topology, homological algebra algebraic geometry. These new developments made it possible for great breakthroughs in solving several difficult problems that were stuck. For example the Deligne's proof of Weil conjectures, Falting's proof of Mordell conjecture and Wile's proof of Fermat's theorem could not have been done in the 19th century just because mathematics was not developed enough. Mathematics of the 20th century has started the path for harmonizing and unifying diverse fields. The unification of mathematics started with a common language that has greatly simplified the interaction between mathematicians. This language became the basis for development of new technical tools for the solution of old problems and the formulation of research programme. As a consequence of the new approach to mathematics, pure mathematicians drifted away from applications and saw no need to collaborate with other scientists, even their traditional neighbors and the physicists. On the other hand, application of the highly abstract modern mathematics could not be easily visualized by the traditional users of mathematics. The period 1930's to 1970's saw a divergence within mathematics itself and between mathematics and other applied sciences. Mathematics became more inward looking and the distinction between pure and applied mathematics became much more pronounced. The diversification of mathematics was first of all connected with external social phenomenon, the rapid growth of the scientific community and the breaking discoveries in Physics. The traditional area of application of mathematics is physics. Within this area the deepest mathematics and success stories have been achieved. For example, Einstein's general theory of relativity was based on classical differential geometry of Riemannian spaces, the Hilbert spaces, the theory of linear operators and spectral theory, In 1930's the connection of mathematics and other sciences, especially physics was broken. Physicists got interacted in solving more concrete problems that could be solved without the application of sophisticated and abstract modern mathematics. The developments of pure mathematics in the post world war II period became weakly connected with applied sciences especially physics. Mathematicians could not view how physics could assist modern mathematics while physicist could not imagine the application of new abstract mathematical concepts such as sheaf, cohomolgy, J-function and the like in their fields.

Re-convergence of Mathematics with Physics:

From the beginning of 1970's, there was a return to mo5re classical topics but on a new level. These developments resulted in the new convergence between mathematics and physics. Some modern mathematicians quickly saw new opportunities and challenges hidden in the new physics. Examples of mathematical results that got inspired by physical ideas include Donaldson's proof of the existence of different structures on simply connected 4-dimensional manifolds. This has very deep consequences for quantum gravity and the gauge theory on strong and weak interactions and resulted in the revisit of the Yang-Mills equations of elementary particles, which had been developed by physicists C.N.Yang and R. Mils almost twenty years earlier in 1954. The Yang-Mils equation had been considered non-physical and had attracted very little attention of Physicists. Structures in the elementary particles are described by highly nonlinear equations with deep topological properties. Donaldson's proof inspired Physicists to a deeper study of the Yang-Mils equations. In 1970's information flow between mathematicians and Physicists resumed and led to new union, theoretical Physicists have made substantial progress in uncovering the principles governing particle interaction. The new conservation laws developed in the last part of the 20^{th} century are believed to be the most fundamental in Physics. Most success stories of application of pure, most abstract mathematics are in Physics. The application of modern abstract Mathematics in physics has resulted in astounding discoveries of the 20^{th} century in the physical sciences, the life sciences and technology. The new approach to mathematics resulted in a more developed mathematical language, new powerful mathematical tools and inspired new application areas that have resulted in tremendous discoveries in other applied sciences including computer science and computer technology. The new mathematical tools and the developments in computer technology, the developments of algorithms, mathematical modeling and scientific computing have led to remarkable new discoveries in physics, technology, economics and other sciences in the last half of the 20^{th} century. This has also enabled mathematicians to use modern mathematical tools to solve deep classical problems left by the previous generation of mathematicians.

New Application Areas:

The branch of mathematics traditionally used in the applications in physics is analysis and differential geometry. Most of the advances in pure mathematics were propelled by problems in physics. In the last part of the 20^{th} century researchers in many other sciences have come to a point where they need serious mathematical tools. The tools of mathematical analysis and differential geometry were no longer adequate. For example a biologist trying to understand the genetic code is discrete. Issues of information content, redundancy or stability of the code are more likely to find tools of theoretical computer science useful than those of classical mathematics are. Even in physics discrete systems such as elementary particles need use of combinatorial tools and statistical mechanics need tools of graph theory and probability theory. Traditionally economics is a heavy user of applied mathematics toolbox. Now economics utilizes sophisticated mathematics in operations research such as linear programming, integer programming, integer programming and other combinatorial optimization models.

Inter – Discipline Mathematics:

Currently, efforts are being undertaken to facilitate collaborative research across traditional academic fields and to help train a new generation of interdisciplinary mathematicians and scientists. Also similar efforts are slowly being introduced in undergraduate and post graduate mathematics curricula and pedagogy. Disciplines that hitherto hardly used mathematics in their curricula are now demanding substantial doses of knowledge of and skills in mathematics. For example the p0re-requisites for mathematical knowledge and skills for entry in into biological and other life sciences as well as the mathematics content in the university curricula of these programmes is becoming quite substantial. Curriculam for the social sciences programmes now include sophisticated mathematics over and above the traditional descriptive statistics. Curricula of some universities in the developed countries have inter-disciplinary programmes where mathematics students and students from other sciences (including social sciences) work jointly on projects. The aim is to prepare graduates for the new approaches and practices in their fields and careers.

Mathematics in Digital Technology:

The mathematics of multimedia encompasses a wide range of research areas, which include computer vision, image processing, speech recognition and language understanding, computer aided design, and new modes of networking. The mathematical tools in multimedia may include stochastic processes, Markov fields, statistical patterns, decision theory, PDF, numerical an analysis, graph theory, graphic algorithms, image analysis and wavelets, and many others. Computer aided design is becoming a powerful tool in many industries. This technology is a potential area for research mathematicians. The future of the world wide web (www) will depend on the development of many new mathematical ideas and algorithms, and mathematicians will have to develop ever more secure cryptographic schemes and thus new developments from number theory, discrete mathematics, algebraic geometry, and dynamical systems, as well as other fields.

Mathematics in the Army:

Recent trends in mathematics research in the USA Army have been influenced by lessons learnt during combat in Bosnia. The USA army could not bring heavy tanks in time and helicopters were not u8sed to avoid casualty. Also there is need for lighter systems with same or improved requirement as before. Breakthroughs are urgently needed and mathematics research is being funded with a hope to get the urgently needed systems. These future automated systems are complex and non- linear; they will likely be multiple units, small in size, light in weight, very efficient in energy utilization and extremely fast in speed and will likely be self organized and self coordinated to perform special tasks. Research areas are many and exciting. They include:(i) Mathematics for materials (Materials by design – optimization on microstructures; Energy source-compact power, Energy efficiency: Nonlinear Dynamics and Optimal Control) (ii) Security issues (needs in critical infrastructure protection, mathematics for information and Communication, Mathematics for sensors, i.e. information/data mining and fusion, information on the move i.e. mobile communication as well as network security and protection) (iii) Demands in software reliability where mathematics is needed for computer language, architecture, etc. (iv) Requirements for automated decision making (probability, stochastic analysis, mathematics of sensing, pattern analysis, and spectral analysis) and (v) Future systems (lighter vehicles, smaller satellites, ICBM Interceptors, Hit before being Hit, secured wireless communication systems, super efficient energy/power sources, modeling and simulations, robotics and automation. During the last years, developments in mathematics, in computing and communication technologies have made it possible for moist of the breath taking discoveries in basic sciences, for the tremendous innovations and inventions in engineering sciences and technology and for the great achievements and breakthroughs in economics and life sciences. These have led to the emergency of many new areas of mathematics and enabled areas that were dormant to explode. Now every branch of mathematics has a potential for applicability in other fields of mathematics and other disciplines. All these, have posed a big challenge on the mathematics curricula at all levels of the education systems, teacher preparation and pedagogy. The 21st Century mathematics thinking is to further strengthen efforts to bridge the division lines within mathematics, to open up more for other disciplines and to foster the line of inter-discipline research.

Conclusions and recommendations:

Finally we would like to say that modeling should be given a more prominent role in physics education, based on the nature of physics and on research on the teaching and learning of physics. We have suggested the framework with multiple representations of physical phenomena as a good basis for developing meaningful learning activities for students in a physics course focused on modeling. Moreover, we have described modeling exercises used in the paper and looked briefly at the assessment of students' modeling competency.

A main conclusion from the achievement test and student questionnaire is that students' understanding of the nature of science, their learning strategies and their competency to handle multiple representations appear to reinforce each other. In particular, we believe that students who have acquired appropriate learning strategies are more able to 'decode the language of physics' (i.e. the use of multiple representations during physics lessons) and hence bring the teacher's and textbook's use of representations to fruition (the result that you wanted to achieve from a plan or idea) in their own processing and learning of physics. We therefore suggest that the empirical-mathematical modeling approach and the focus on representations is worth elaborating further, but that it needs to be accompanied by explicit but integrated attention to the nature of science and students' learning strategies.

REFERENCES:

1 Monastyrsky, Michael, "some trends in Modern Mathematics and the Fields Medal", Notes-de la SMC, Volume 33, Nos.2 and 3.

2. Mumford, David, Trends in the profession of Mathematics: Choosing our Directions", Berlin Intelligencer

3. Hoyningen-Huene,p.,weber M.and Oberhem, E.(1999), "Science for the 21st century: A New commitment", Background paper presented at The World Conference on Science, 26th June - 1st July 1999 ,Budapest.

4. Avner Friedman, Reflections on the future of Mathematics: Wes are on the threshold of ever esxpandin g horizons",Berlin Intelligencer, ICM August

5. Lovasz, Laszlo, 1998, "One Mathematics: There is no natural way to divide mathematics", Berlin Intelligencer, ICM August 1998

6. Jaffe, A., Baouendi, S., and Lipman, J" Demotion of mathematics meets groundswell of Protest "

7. Hu, J.J and Wang, H.,2001, "US national research trends in mathematics and intelligent Control: Trends in DOD/Army Funding in Mathematics Research".