On $w - \delta - \mathfrak{T}$ −Open Sets

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Abstract. *In this paper we will introduce $w - \delta - \mathfrak{T} -$ open sets and discuss some properties and characterizations of it. Also Examples are given throughout the paper.*

Key Words and phrase . $w - \alpha - \mathfrak{T} -$ open , w - semi – $\mathfrak{T}$ - open , w – pre – $\mathfrak{T}$ - open , $w - \beta - \mathfrak{T} -$open , $w - \delta - \mathfrak{T} -$open

1. Introduction

The subject of ideals in topological spaces were introduced by Kuratowski [3] and further studied by Vaidyanathaswamy [5]. Corresponding to an ideal a new topology $\tau^*(\mathfrak{T}, \tau)$ called the * - topology was given which is generally finer than the original topology having the kuratowski closure operator $cl^*(A) = A \cup A^*(\mathfrak{T}, \tau)$ [6] , where $A^*(\mathfrak{T}, \tau) = \{x \in X : U \cap A \notin \mathfrak{T}$ for every open subset U of x in X } called a local function of A with respect to $\mathfrak{T}$ and τ. We will write τ^* for $\tau^*(\mathfrak{T}, \tau)$.

The following section contains some definitions and results that will be used in our further sections.

Definition 1.1.[3] Let (X, τ) be a topological space. An ideal $\mathfrak{T}$ on X is a collection of non-empty subsets of X such that (a) $\emptyset \in \mathfrak{T}$ (b) $A \in \mathfrak{T}$ and $B \in \mathfrak{T}$ implies $A \cup B \in \mathfrak{T}$ (c) $B \in \mathfrak{T}$ and $A \subset B$ implies $A \in \mathfrak{T}$.

Definition 1.2.[1] Let $(X, \tau, \mathfrak{T})$ be an ideal space and A be any subset of X. Then A is said to be δ – $\mathfrak{T}$ – open if $int(cl^*(A)) \subset cl^*\big(int(A)\big)$.

Definition 1.3. [4] Let $(X, \tau, \mathfrak{T})$ be an ideal space and A be any subset of X. Then A is said to be

a.) $w - \alpha - \mathfrak{T} -$ open if $A \subset int^*\Big(cl\big(int^*(A)\big)\Big)$.
b.) $w -$ semi – $\mathfrak{T}$ - open if $A \subset cl\big(int^*(A)\big)$.
c.) $w -$ pre – $\mathfrak{T}$ - open if $A \subset int^*\big(cl(A)\big)$.
d.) $w - \beta - \mathfrak{T} -$ open if $A \subset cl\Big(int^*\big(cl(A)\big)\Big)$.

2. Results

Definition 2.1 . Let $(X, \tau, \mathfrak{T})$ be an ideal topological space. Then a subset A of X is called $w - \delta - \mathfrak{T} -$ open if $int^*(cl(A)) \subset cl(int^*(A))$ and a subset A of X is called $w - \delta - \mathfrak{T} -$ closed if its complement is open.

The following examples show that there is no relationship between $w - \delta - \mathfrak{T} -$ open sets and $\delta - \mathfrak{T} -$ open sets.

Example 2.1 . Let $X = \{ a, b, c \}$ with $\tau = \{ \emptyset , \{a, b \}, X \}$ and an ideal $\mathfrak{T} = \{ \emptyset , \{a\} \}$

So, $\tau^* = \{ \emptyset, \{b\}, \{a, b\}, \{b, c\}, X \}$

Consider the subset $A = \{ b, c \}$

Then, $int(cl^*(A)) = int (\{ a, b, c \})$

$= \{ a, b, c \}$

and $cl^*(int(A)) = cl^* (\emptyset) = \emptyset$

$\therefore \ int(cl^*(A)) \not\subset cl^*(int(A))$

$\Rightarrow A$ is not $\delta - \mathfrak{T} -$ open subset of X .

But, $int^*(cl(A)) = int^* (\{a, b, c \})$

$= \{a, b, c \}$

and $cl(int^*(A)) = cl (\{ b, c \})$

$= \{a, b, c \}$

$\therefore \ int^*(cl(A)) \subset cl(int^*(A))$

$\Rightarrow \ A$ is $w - \delta - \mathfrak{T} -$ open subset of X.

Hence, $w - \delta - \mathfrak{T} -$ open $\not\Rightarrow$ $\delta - \mathfrak{T} -$ open

Example 2.2 . Let $X = \{ a, b, c \}$ with $\tau = \{ \emptyset , \{a, b \}, X \}$ and an ideal $\mathfrak{T} = \{ \emptyset , \{a\} \}$

So, $\tau^* = \{ \emptyset, \{b\}, \{a, b\}, \{b, c\}, X \}$

Consider the subset $A = \{ a, c \}$

Then, $int(cl^*(A)) = int (\{ a, \ c \})$

$= \emptyset$

and $cl^*(int(A)) = cl^* (\emptyset) = \emptyset$

$\therefore \ int(cl^*(A)) \subset cl^*(int(A))$

$\Rightarrow A$ is $\delta - \mathfrak{T} -$ open subset of X .

But, $int^*(cl(A)) = int^*(\{a, b, c\})$

$= \{a, b, c\}$

and $cl(int^*(A)) = cl(\emptyset)$

$= \emptyset$

$\therefore \ int^*(cl(A)) \not\subset cl(int^*(A))$

$\Rightarrow A$ is not $w - \delta - \mathfrak{T} -$ open subset of X.

Hence, $\delta - \mathfrak{T} -$ open $\not\Rightarrow w - \delta - \mathfrak{T} -$ open

Proposition 2.1 . Let $(X, \tau, \mathfrak{T})$ be an ideal topological space, then a subset of X is $w -$ semi $- \mathfrak{T} -$ open if and only if it is both $w - \delta - \mathfrak{T} -$ open and $w - \beta - \mathfrak{T} -$ open .

Proof . Firstly, Let A be $w -$ semi $- \mathfrak{T} -$ open subset of X.

Then , $A \subset cl(int^*(A))$

$\Rightarrow A \subset cl\left(int^*(cl(A))\right)$

$\Rightarrow A$ is $w - \beta - \mathfrak{T} -$ open subset of X.

Also, $int^*(cl(A)) \subset cl(A) \subset cl\left(cl(int^*(A))\right)$

$= cl(int^*(A))$

$\Rightarrow int^*(cl(A)) \subset cl(int^*(A))$

$\therefore A$ is $w - \delta - \mathfrak{T} -$ open subset of X.

Conversely .

Let A be $w - \delta - \mathfrak{T} -$ open and $w - \beta - \mathfrak{T} -$ open subset of X.

This implies that

$int^*(cl(A)) \subset cl(int^*(A))$ ....(1)

and $A \subset cl\left(int^*(cl(A))\right)$ ....(2)

Now, $A \subset cl\left(int^*(cl(A))\right)$

$\subset cl\left(cl(int^*(A))\right)$ Using (1)

$= cl(int^*A)$

Hence, A is $w -$ semi $- \mathfrak{T} -$ open subset of X.

The following example shows that there is no relationship between $w-\delta-\mathfrak{I}-$ open sets and $w-\beta-\mathfrak{I}-$ open sets.

Example 2.3 . In Example 2.1, if we take the subset $A=\{a\}$

Then,

$int^*(cl(A)) = int^*(\{a,b,c\})$

$= \{a,b,c\}$

and $cl(int^*(A)) = cl(\emptyset)$

$= \emptyset$

$\therefore\ int^*(cl(A)) \not\subset cl(int^*(A))$

$\Rightarrow$ A is not $w-\delta-\mathfrak{I}-$ open subset of X.

But, $cl\left(int^*(cl(A))\right) = cl(\{a,b,c\} = \{a,b,c\}$

$\Rightarrow A \subset cl\left(int^*(cl(A))\right)$

$\therefore$ A is $w-\beta-\mathfrak{I}-$ open subset of X.

Hence, $w-\beta-\mathfrak{I}-$ open $\not\Rightarrow$ $w-\delta-\mathfrak{I}-$ open.

Also, for the subset $A=\{c\}$

$int^*(cl(A)) = int^*(\{c\})$

$= \emptyset$

and $cl\left(int^*(A)\right) = cl(\emptyset)$

$= \emptyset$

$\Rightarrow$ $int^*(cl(A)) \subset cl\left(int^*(A)\right)$

$\Rightarrow$ A is $w-\delta-\mathfrak{I}-$ open subset of X.

But $cl\left(int^*(cl(A))\right) = cl(\emptyset) = \emptyset$

$\Rightarrow$ A is not $w-\beta-\mathfrak{I}-$ open subset of X.

Hence, $w-\delta-\mathfrak{I}-$ open $\not\Rightarrow$ $w-\beta-\mathfrak{I}-$ open .

Proposition 2.2 . Let $(X,\tau,\mathfrak{I})$ be an ideal topological space. Then every τ^*- open is $w-\delta-\mathfrak{I}-$ open subset of X .

Proof . Let G be $\tau^* -$ open subset of X .

Then, $G = int^* (G)$ .... (1)

Since, $int^*\big(cl(G)\big) \subset cl\,(G)$

$= cl\,\big(\,int^*\,(G)\big)$ Using (1)

So, $int^*\big(cl(G)\big) \subset cl\,\big(\,int^*\,(G)\big)$

$\Rightarrow$ every $\tau^* -$ open subset of X is $w - \delta - \mathfrak{T} -$ open.

Proposition 2.3 . Let $(\,X, \tau, \mathfrak{T}\,)$ be an ideal topological space. Then a subset of X is $w - \alpha - \mathfrak{T} -$ open if and only if it is both $w - \delta - \mathfrak{T} -$ open and $w -$ pre $- \mathfrak{T} -$ open .

Proof . Firstly, Let A be $w - \alpha - \mathfrak{T} -$ open subset of X.

Then , $A \subset int^*\Big(cl\big(int^*(A)\big)\Big)$

$\subset cl\,\big(\,int^*(A)\big)$

$\Rightarrow A$ is $w -$ semi $- \mathfrak{T} -$ open .

$\therefore$ by proposition 2.1,

A is $w - \delta - \mathfrak{T} -$ open subset of X.

Also, $A \subset int^*\Big(cl\big(int^*(A)\big)\Big)$

$\subset int^*\big(cl(A)\big)$ since, $int^*(A) \subset A$

$\Rightarrow A$ is $w -$ pre $- \mathfrak{T} -$ open subset of X.

Conversely.

Let, A be $w - \delta - \mathfrak{T} -$ open and $w -$ pre $- \mathfrak{T} -$ open subset of X .

Then , $int^*\big(cl(A)\big) \subset cl\big(int^*(A)\big)$

$\Rightarrow int^*\Big(int^*\big(cl(A)\big)\Big) \subset int^*\Big(cl\big(int^*(A)\big)\Big)$

$\Rightarrow int * \big(cl(A)\big) \subset int^*\Big(cl\big(int^*(A)\big)\Big)$

Further, A is $w -$ pre $- \mathfrak{T} -$ open subset of X .

So, $A \subset int^*\big(cl(A)\big)$

$\Rightarrow A \subset int^*\big(cl(A)\big) \subset int^*\Big(cl\big(int^*(A)\big)\Big)$

$\Rightarrow A$ is $w - \alpha - \mathfrak{T} -$ open subset of X.

Proposition 2.4 . Let $(X, \tau, \mathfrak{I})$ be an ideal topological space and A, B be two subsets of X such that $A \subset B \subset cl(A)$. If A is $w - \delta - \mathfrak{I} -$ open then B is $w - \delta - \mathfrak{I} -$ open subset of X.

Proof . Let $A \subset B \subset cl(A)$ and A be $w - \delta - \mathfrak{I} -$ open subset of X.

Then, $int^*\big(cl(A)\big) \subset cl\left(int^*(A)\right)$

Now, $A \subset B$

$\Rightarrow cl\big(int^*(A)\big) \subset cl\big(int^*(B)\big)$

Also, $B \subset cl(A)$

$\Rightarrow cl(B) \subset cl\big(cl(A)\big)$

$= cl(A)$

$\Rightarrow int^*\big(cl(B)\big) \subset int^*\left(cl(A)\right)$

$\Rightarrow int^*\big(cl(B)\big) \subset int^*\left(cl(A)\right) \subset cl\big(int^*(B)\big)$

$\therefore int^*\big(cl(B)\big) \subset cl\big(int^*(B)\big)$

Hence, B is $w - \delta - \mathfrak{I} -$ open subset of X.

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