Study of Triangular Solution for The (ERTBP) Elliptic Restricted Three Body Problem under Radiating and Oblate Primaries

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Abstract

In this paper, The elliptic restricted three- body problem (ERTBP) has been discussed in this paper the stability of triangular equilibrium points($L_{4,5}$) is studied when both oblate primaries emit light energy simultaneously using pulsating coordinatessystem. It has been found that the region of stability increases and decreases with variability in oblateness, eccentricity and radiation pressure. Due to the eccentricity, semi-major axis, radiation and oblateness factors of both primaries the position of the triangular points are seen to be shift away from the line joining the primaries than in the classical case. The computed model under the location of the triangular points has been considered. New expression for the location of the triangular equilibrium points in terms of power series has been obtained for the mass ratio parameter. The graph has been plotted against these location and for the whole domain of the mass ratio for different perturbation. The linearized equation of motion are found near the triangular equilibrium points and discussed about the linear stability of the triangular points. It was found that the stability depends on eccentricity, oblateness coefficient and radiation pressure. The stable regionfor the binary stars are obtained.

Keywords: Elliptical Restricted Three Body Problem; Stability; Radiation; Oblateness; Binary System.

1. Introduction

The Circular Restricted Three Body Problem(CRTBP), is defined as a dynamical system in which m_1 and m_2 are two massive bodies called primaries which moves on circular orbits around their common barycenter assumed on same plane where m_3 is an infinitesimal mass moving in this field. The primaries m_3 can be safely neglected since it is assumed that its mass is small.(Szebehely-1967)The restricted three-body problem has received much attention with or without perturbing forces as it is very rich dynamical system. The equations of motion are, presented in a non-inertial coordinate system that rotates with the mean motion of the primaries (Murray and Dermott 1999). In the rotating co ordinate system, the positions of the primaries are kept fixed. The inability to treat long time behaviour of CRTBP in Celestial mechanics is the major fault. The principal reason behind this is that the significant effects might be expected because of the eccentricity of the orbit of primaries ,see Singh and Umar(2012).A non uniformly pulsating coordinate system is commonly used when the primaries orbit is elliptical rather than circular. These new coordinates have the felicitous property that, the positions of the primaries are kept fixed and the Hamiltonian does not explicitly depend on time, see Szebehely (1967).

In actual situations most of celestial bodies are oblate spheroids but the participating bodies in the CRTBP are strictly spherical in shape. The large perturbations on the two-body is caused due to lack of sphericity, or the oblateness, of the planet or star. An equatorial bulge which is oblateness comes from the centrifugal force due to rotation. Most

of celestial bodies are sufficiently oblate to make the departure from sphericity. The literature is wealth with works dealing with the circular as well as elliptic restricted three body problem with or without the oblateness and triaxial perturbations, and/or radiating sources. E.g. Tsirogiannis et al. (2006), Sharma (1987), Vishnu Namboori et al. (2008), Mital et al. (2009), Ishwar and Kushvah (2006), AbdulRaheem and Singh (2006, 2008), Singh and Ishwar (1999), Kumar and Ishwar (2009), all of these authors treated the restricted three body problem when one or both primaries as sources of radiation or oblate spheroids or both.

Some work done in this field of ERTBP:Singh and Umar (2012) studied the motion of an infinitesimal mass around seven equilibrium points in the framework of the ERTBP under the assumption that the primary of the system is a non-luminous, oblate spheroid and the secondary is luminous. They studied a motion of the dynamical evolution of dust particles in orbits around a binary system with a dark degenerate primary and a secondary stellar companion and Concluded that due to this effect the size of the region of stability decreases when the value of these parameters increases. The out-of-plane equilibrium points and the collinear points are found to be unstable for any combination of the parameters considered here.SubbaRao and Sharma (1975) considered the primary as an oblate spheroid, whose equatorial plane coincides with the plane of motion, they proved that the stable solutions decreases due to oblateness because of the range of the mass parameter.Elipe and Ferrer (1985) considered both primaries as triaxial rigid bodies with one of the axes as axis of symmetry and its equatorial plane coinciding with the plane of motion. They examined three rigid bodies under central forces in the CRTBP and obtained collinear and triangular solutions. Sharma et al. (2001) discussed that collinear points are unstable, whereas the triangular points are conditionally stable and described the stability of equilibrium points. Ammar (2008) studied the effect of solar radiation pressure on the location and stability of the five Lagrangian points, within the frame of ERTBP. He obtained new formulas for the location of the collinear libration points and concluded that the radiation pressure plays the rule of slightly reducing the effective mass of the Sun and changes the location of the Lagrangian points. Usha et al. (2014) studied the motion of an infinitesimal mass around triangular equilibrium points in the ERTBP assuming bigger primary as a source of radiation and the smaller one a triaxial rigid body. They found that the critical mass ratio depends on the oblateness, radiation pressure, eccentricity and semi major axis of the elliptic orbits and the range of radiation parameter increases with decreases in stability. Narayan and Usha (2014) investigated that in the ERTBP the stability of infinitesimal motions about the triangular equilibrium points assuming bigger primary as a source of radiation and the smaller one a triaxial rigid body. Singh and Umar (2014) examined the motion of a dust grain around a triaxial primary and an oblate companion orbiting each other in elliptic orbits about their common barycenter in the neighbourhood of collinear libration points. The positions and stability of these points are found to be affected by the triaxiality and oblateness of the primaries, and by the semi-major axis and eccentricity of their orbits.

The present work aims to determine the locations of the triangular equilibrium points and to investigate their stability in the ERTBP when both primaries emit light energy simultaneously using pulsating coordinates. This paper is organized as: Sect.1 ,Introduction; Sect. 2 provides the equation of motion and locations of triangular equilibrium points and its numerical application with graphical representation ; Sect.3 the linear stability of the triangular points Sect. 4 Analysis of stability and instability region. Conclusions are drawn in Sect.5

2.Equation of motion

Since the orbits of the primaries are elliptic, in order to maintain the primaries in fixed positions-the rotating frame of reference must be a system which rotates uniformly with axes which expand and shrink, the equations of motion of the infinitesimal mass are presented here in dimensionless units in such a rotating pulsating coordinate system following Singh and Umar (2012) as:

$$\begin{aligned} \xi'' - 2\eta' &= U_{\xi}^{*} \qquad \eta'' + 2\xi' = U_{\eta}^{*} \qquad \zeta'' = U_{\zeta}^{*} \qquad (2.1) \\ U &= \left(1 - e^{2}\right)^{-\frac{1}{2}} \left[\frac{\xi^{2} + \eta^{2}}{2} + \frac{1}{n^{2}} \left(\frac{(1 - \mu)q_{1}}{r_{1}} + \frac{\mu q_{2}}{r_{2}} + \frac{(1 - \mu)A_{1}q_{1}}{2r_{1}^{3}} + \frac{\mu A_{2}q_{2}}{2r_{2}^{3}}\right)\right] \end{aligned}$$

The mean motion ,n ,is given by

$$n^{2} = \frac{\left(1+e^{2}\right)^{\frac{1}{2}}\left(1+\frac{3}{2}A_{1}+\frac{3}{2}A_{2}\right)}{a\left(1-e^{2}\right)}$$
(2.3)

$$r_i^2 = (\xi + \xi_i)^2 + \eta^2 + \zeta^2 \quad \xi_1 = \mu \ \xi_2 = 1 - \mu \qquad \mu = \frac{m_2}{m_1 + m_2}$$
(2.4)

Here, m_1 , m_2 are the masses of the bigger and smaller Primaries positioned at the points ($\xi_{i,0}$, 0), $i = 1, 2; q_1, q_2$ are their mass reduction factors; A_1 , A_2 are their oblateness coefficients; r_i , (i = 1, 2) are the distances of the infinitesimal mass from the bigger and smaller primaries, respectively; while *a* and *e*are respectively the semi-major axis and eccentricity of the orbits.

Locations of triangular equilibrium points

The position of equilibrium points can be found by setting all relative velocity and relative acceleration components equal to zero and solving the resulting system,

$$U_{\xi} = U_{\eta} = U_{\zeta} = 0 \tag{2.5}$$

where

$$\begin{split} \xi &- \frac{1}{n^2} \left[\frac{(1-\mu)q_1(\xi+\mu)}{r_1^3} + \frac{\mu q_2(\xi+\mu-1)}{r_2^3} + \frac{3(1-\mu)A_1q_1(\xi+\mu)}{2r_1^5} + \frac{3\mu A_2q_2(\xi+\mu-1)}{2r_2^3} \right] = 0 \\ \eta \left[1 - \frac{1}{n^2} \left\{ \frac{(1-\mu)q_1}{r_1^3} + \frac{\mu q_2}{r_2^3} + \frac{3(1-\mu)A_1q_1}{2r_1^5} + \frac{3\mu A_2q_2}{2r_2^3} \right\} \right] = 0 \\ \zeta \left[\frac{(1-\mu)q_1}{r_1^3} + \frac{\mu q_2}{r_2^3} + \frac{3(1-\mu)A_1q_1}{2r_1^5} + \frac{3\mu A_2q_2}{2r_2^3} \right] = 0 \end{split}$$

The solution of the first two equation of system (2.5) with $\eta \neq 0$, $\zeta=0$ gives the positions of the triangular points from which we obtain

$$\xi n^{2} - \frac{(1-\mu)q_{1}(\xi+\mu)}{r_{1}^{3}} - \frac{\mu q_{2}(\xi+\mu-1)}{r_{2}^{3}} - \frac{3(1-\mu)A_{1}q_{1}(\xi+\mu)}{2r_{1}^{5}} - \frac{3\mu A_{2}q_{2}(\xi+\mu-1)}{2r_{2}^{3}} = 0$$

$$n^{2}\eta - \eta \left\{ \frac{(1-\mu)q_{1}}{r_{1}^{3}} + \frac{\mu q_{2}}{r_{2}^{3}} + \frac{3(1-\mu)A_{1}q_{1}}{2r_{1}^{5}} + \frac{3\mu A_{2}q_{2}}{2r_{2}^{3}} \right\} = 0$$
(2.6)
(2.7)

Since the oblateness and radiation coefficients are very small, i.e. A_1, A_2 , i = 1, 2 therefore Ignoring these perturbations gives the equilateral solution of the classical restricted three-body problem i.e. $r_1 = r_2 = 1$, then it may be reasonable in our case to assume that the positions of the equilibrium points $L_{4,5}$ are the same as given by classical restricted three-body problem but perturbed by terms factored by $\varepsilon_{1,2} \equiv O(A_1, A_2)$.

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$$r_{1} = 1 + \varepsilon_{1}, \qquad r_{2} = 1 + \varepsilon_{2} \qquad (2.8)$$

$$(2.6) \text{ into } (2.7) \text{ and solving for } \zeta \text{ and } \eta \text{ up to the first order we get}$$

$$\zeta = \frac{1}{2} - 2\mu - \frac{3}{2}\varepsilon_{1} - \frac{3}{2}A_{1} - 3\mu\varepsilon_{2}$$

$$\eta = \pm \frac{\sqrt{3}}{2} \left[1 + \frac{7}{3}\varepsilon_{1} + \frac{2}{3}q_{1} + \frac{A_{1}}{2} - 2\mu\varepsilon_{2} \right] \qquad (2.9)$$

Substituting the values of r_1 , r_2 , ξ and η into Eqs. (2.6) and (2.7), and expanding the resulting equations, ignoring all the higher order terms in ε_1 and ε_2 and the first order, as well as the mixed terms. Thus we get the following two simultaneous equations in ε_1 , ε_2

$$B_1\varepsilon_1 + C_1\varepsilon_2 + S_1 = 0$$
$$B_2\varepsilon_1 + C_2\varepsilon_2 + S_2 = 0$$
We have

Where

$$B_{1} = \frac{1}{4} [6 + 15A_{1}] + \frac{\mu}{4} [-18 - 14A_{1} + 9A_{2}]$$

$$C_{1} = -\frac{\mu}{4} [6 - 3A_{1} - 15A_{2}]$$

$$S_{1} = -\frac{3}{2}A_{1} + \frac{3\mu}{4} (3A_{1} + A_{2})$$

$$B_{2} = \frac{\sqrt{3}}{4} [(10 + 21A_{1}) + \mu(-10 - 28A_{1} + 7A_{2})]$$

$$C_{2} = \frac{\sqrt{3}\mu}{4} [10 + 6A_{1} - 25A_{2}]$$

$$S_{2} = \frac{\sqrt{3}}{4} [-10A_{1} + \mu(-20A_{1} + 5A_{2})]$$

solving simultaneous equations for ε_1 , ε_2 we get

$$\varepsilon_{1} = -\frac{1}{n^{2}} \frac{S_{1}C_{2} - S_{2}C_{1}}{B_{1}C_{2} - B_{2}C_{1}} \qquad B_{1}C_{2} - B_{2}C_{1} \neq 0$$

$$\varepsilon_{2} = -\frac{1}{n^{2}} \frac{S_{1}B_{2} - S_{2}B_{1}}{C_{1}B_{2} - C_{2}B_{1}} \qquad C_{1}B_{2} - C_{2}B_{1} \neq 0$$

Which can also be written as a function up to order 3 in μ as

$$\varepsilon_1 = \frac{1}{n^2} \sum_{k=0}^4 D_{1,k} \mu^k$$
, $\varepsilon_2 = \frac{1}{n^2} \sum_{k=0}^4 D_{2,k} \mu^k$

Where the non-vanishing coefficient $D_{1,k}$ and $D_{2,k}$ are given in the Appendix.

Substituting the values of ε_1 , ε_2 into Eq. (9) yields the coordinates of the triangular points

$$\begin{aligned} \xi_{L_{4,5}} &= \frac{1}{2} - 2\mu - \frac{3}{2}A_1 - \frac{3}{2n^2} \left[\frac{S_1(2\mu B_2 - C_2) - S_2(2\mu B_1 - C_1)}{B_1 C_2 - B_2 C_1} \right] \\ B_1 C_2 - B_2 C_1 &\neq 0 (2.10) \\ \eta_{L_{4,5}} &= \frac{\sqrt{3}}{2} \left[1 + \frac{2}{3}q_1 + \frac{A_1}{2} - \frac{1}{2n^2} \left[\frac{S_1(7C_2 + 4\mu B_2) - S_2(7C_1 + 4\mu B_1)}{B_1 C_2 - B_2 C_1} \right] \right] \\ B_1 C_2 - B_2 C_1 &\neq 0 (2.11) \end{aligned}$$

Equation (2.10) and (2.11) can be written in more compact form as an ordered pair after soon lengthy algebraic calculation, the location of $L_{4,5}$ are:

$$\left(\xi_{L_{4,5}},\eta_{L_{4,5}}\right) = \left(\frac{1}{2} - 2\mu - \frac{3}{2}A + \frac{3}{2n^2}\sum_{K=0}^{5}T_k\mu^k, \pm\frac{\sqrt{3}}{2} + \frac{A_1\sqrt{3}}{4} + \frac{q_1}{\sqrt{3}} + \frac{1}{2n^2}\sum_{K=0}^{5}T_k\mu^k\right)$$

Where the non-vanishing coefficient T_k, are

$$T_{0} = 180A_{1} - 30A_{1}^{2} + 400A_{1}A_{2} - 423A_{2}^{2}$$

$$T_{1} = 380A_{1} - 90A_{2} - \frac{3655}{2}A_{1}A_{2} + 948A_{1}^{2} - 200A_{2}^{2}$$

$$T_{2} = -710A_{1} - 10A_{2} - \frac{5403}{2}A_{1}A_{2} - \frac{9009}{2}A_{1}^{2} - 827A_{2}^{2}$$

$$T_{4} = -1500A_{1} + 340A_{2} + 6207A_{1}A_{2} - 10110A_{1}^{2} - 843A_{2}^{2}$$

Graphical representation

In fig 1-3 we have plotted for different values of oblateness coefficientdescribed as folloes 1) zero value of both the oblateness yield fig.5 ,2) small value of $A_{1=}$.0001 and A_2 .001 in Fig -6,3) changed value of $A_{1=}$.001 and A_2 .0001 in Fig -7 and the value of q=.9997 and observed that oblateness dominate perturbation.



Fig-3







3. Linear stability of the triangular points

The position of the motion of the infinitesimal body is displaced a little from the equilibrium point due to the included perturbations. If the resultant motion of the infinitesimal mass is a rapid departure from the vicinity of the point, we can call such a position of equilibrium point an "unstable one", if however the body merely oscillates about the equilibrium point, it is said to be a "stable position" (in the sense of Lyapunov). In order to analyse the stability, one starts by introducing a displacement (δ_{ξ} , δ_{η}) from the libration points, say $\xi = \xi_{L4,5} + \delta_{\xi}$, $\eta = \eta_{L4,5} + \delta_{\eta}$, where ($\xi_{L4,5}$, $\eta_{L4,5}$) coincides with one of the five stationary solutions; the linearized equation can be written as

$$\delta_{\xi}^{"} - 2\delta_{\eta}^{'} = U_{\xi\xi}^{(L_{4,5})} \delta_{\xi} + U_{\xi\eta}^{(L_{4,5})} \delta_{\eta}$$

$$\delta_{\eta}^{"} - 2\delta_{\xi}^{'} = U_{\eta\xi}^{(L_{4,5})} \delta_{\xi} + U_{\eta\eta}^{(L_{4,5})} \delta_{\eta}$$
(3.1)

where $U_{\xi\xi}^{L_{4,5}}$ denote the second derivative of U with respect to ξ computed at the stationary solution ($\xi_{L4,5}$, $\eta_{L4,5}$) (similarly for the other derivative). The characteristic equation corresponding to (3.1), is

$$\lambda^{4} - (U_{\xi\xi}^{(L_{4,5})} + U_{\eta\eta}^{(L_{4,5})} - 4)\lambda^{2} + U_{\xi\xi}^{(L_{4,5})} U_{\eta\eta}^{(L_{4,5})} - (U_{\xi\eta}^{(L_{4,5})})^{2} = 0$$
(3.2)

First compute the partial derivatives required for equation (3.2) as

$$U_{\xi\xi} = \frac{\left(1-e^{2}\right)^{-\frac{1}{2}}}{n^{2}} \left[n^{2} - \left(1-\mu\right)q_{1} \left\{ \frac{1}{r_{1}^{3}} - \frac{3}{r_{1}^{5}} \left(\xi+\mu\right)^{2} \right\} - \mu q_{2} \left\{ \frac{1}{r_{2}^{3}} - \frac{3}{r_{2}^{5}} \left(\xi+\mu-1\right)^{2} \right\} - \frac{3}{2} \left(1-\mu\right)q_{1}A_{1} \left\{ \frac{1}{r_{1}^{5}} - \frac{15}{2r_{1}^{7}} \left(\xi+\mu\right)^{2} \right\} - \frac{3}{2} \mu q_{2}A_{2} \left\{ \frac{1}{r_{2}^{5}} - \frac{15}{2r_{2}^{7}} \left(\xi+\mu-1\right)^{2} \right\} \right]$$
(3.3)

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$$U_{\eta\eta} = \frac{\left(1-e^{2}\right)^{-\frac{1}{2}}}{n^{2}} \left[n^{2} - \left(1-\mu\right) q_{1} \left\{ \frac{1}{r_{1}^{3}} - \frac{3}{r_{1}^{5}} \eta^{2} \right\} - \mu q_{2} \left\{ \frac{1}{r_{2}^{3}} - \frac{3}{r_{2}^{5}} \eta^{2} \right\} - \frac{3}{2} \left(1-\mu\right) q_{1} A_{1} \left\{ \frac{1}{r_{1}^{5}} - \frac{15}{2r_{1}^{7}} \eta^{2} \right\} - \frac{3}{2} \mu q_{2} A_{2} \left\{ \frac{1}{r_{2}^{5}} - \frac{15}{2r_{2}^{7}} \eta^{2} \right\} \right]$$
(3.4)

$$U_{\xi\eta} = \frac{\left(1 - e^2\right)^{-\frac{1}{2}}}{n^2} \eta \left[\frac{3(1 - \mu)q_1(\xi + \mu)}{r_1^5} - \frac{3\mu q_2(\xi + \mu - 1)}{r_2^5}\right]$$
(3.5)

Let us suppose that $\lambda = i\omega$ in (3.2) the characteristic equation can be written in the form

$$\omega^{4} - u_{2}^{(L_{4,5})} \omega^{2} + u_{0}^{(L_{4,5})} = 0_{\text{with}}$$

$$u_{2}^{(L_{4,5})} = -(U_{\xi\xi}^{(L_{4,5})} + U_{\eta\eta}^{(L_{4,5})} - 4)$$

$$u_{0}^{(L_{4,5})} = U_{\xi\xi}^{(L_{4,5})} U_{\eta\eta}^{(L_{4,5})} - (U_{\xi\eta}^{(L_{4,5})})^{2}$$
(3.6)

The required partial derivatives are obtained using equation (3.3) (3.4) (3.5) after setting the locations of L₄ and L₅ as given by $r_1 = 1 + \varepsilon_1$, $r_2 = 1 + \varepsilon_2$, we have

$$U_{\xi\xi} = \frac{\left(1-e^{2}\right)^{-\frac{1}{2}}}{n^{2}} \Big[n^{2} - (1-\mu)q_{1} \Big\{ (1-3\varepsilon_{1}) - 3(1-5\varepsilon_{1})(\xi+\mu)^{2} \Big\} - \mu q_{2} \Big\{ (1-3\varepsilon_{2}) - 3(1-5\varepsilon_{2})(\xi+\mu-1)^{2} \Big\} \\ - \frac{3}{2}(1-\mu)q_{1}A_{1} \Big\{ (1-5\varepsilon_{1}) - \frac{15}{2}(1-7\varepsilon_{1})(\xi+\mu)^{2} \Big\} - \frac{3}{2}\mu q_{2}A_{2} \Big\{ (1-5\varepsilon_{2}) - \frac{15(1-7\varepsilon_{2})}{2}(\xi+\mu-1)^{2} \Big\} \Big] \\ U_{\eta\eta} = \frac{\left(1-e^{2}\right)^{-\frac{1}{2}}}{n^{2}} \Big[n^{2} - (1-\mu)q_{1} \Big\{ (1-3\varepsilon_{1}) - 3(1-5\varepsilon_{1})\eta^{2} \Big\} - \mu q_{2} \Big\{ (1-3\varepsilon_{2}) - 3(1-5\varepsilon_{2})\eta^{2} \Big\} \Big]$$

$$-\frac{3}{2}(1-\mu)q_{1}A_{1}\left\{(1-5\varepsilon_{1})-\frac{15}{2}(1-7\varepsilon_{1})\eta^{2}\right\}-\frac{3}{2}\mu q_{2}A_{2}\left\{(1-5\varepsilon_{2})-\frac{15(1-7\varepsilon_{2})}{2}\eta^{2}\right\}\right]$$

$$U_{\xi\eta} = \frac{\left(1 - e^2\right)^{-\frac{1}{2}}}{n^2} \eta \left[3(1 - \mu)q_1(1 - 5\varepsilon_1)(\xi + \mu) - 3\mu q_2(1 - 5\varepsilon_2)(\xi + \mu - 1) + \frac{15(1 - \mu)q_1A_1(1 - 7\varepsilon_1)(\xi + \mu)}{2} + \frac{15\mu q_2A_2(1 - 7\varepsilon_2)(\xi + \mu - 1)}{2}\right]$$
(3.7)

As a fast check, removing all perturbations one gets:

$$U_{\xi\xi}^{(L_{4,5})} = \frac{3}{4} \qquad U_{\eta\eta}^{(L_{4,5})} = \frac{9}{4} \qquad U_{\xi\eta}^{(L_{4,5})} = \frac{3\sqrt{3}}{4} (1-2\mu)$$
$$U_{2}^{(L_{4,5})} = 1 \qquad U_{0}^{(L_{4,5})} = \frac{27}{4}\mu - \frac{27}{4}\mu^{2} \qquad (3.8)$$

4. Analysis of stability and instability region

The regions of stability and instability can be investigated from the roots ω 's of Eq. (3.6). These roots ω_1 , ω_2 , ω_3 and ω_4 are plotted against different values of mass ratios, where we have taken $\mu \in [0, 0.07]$. In Fig. 1, the stability region is shifted. This shift is due to the inclusion of the considered perturbations: elliptic orbits of the primaries together with

their radiating ,oblateness and their triaxialities for the binary system ERTBP. We noticed in our case the critical mass ratio is modified, i.e. $\mu_c = 0.065$ compared with the data given below which is very noticeable.

The stability region of binary system



Fig.Stable region for binary star

Table 1

Binary System	Mass	Critical mass value		Oblateness		Locations of	
	ratio					triangular points	
	μ	q1	q2	A1	A2	بح	±η
Achird	0.3949	0.9971	0.9997	0	0	0.10164	1.9211
				.01	.02	.09677	1.8838
				.1	.2	.05302	1.5069
Luyten 726-8	0.5	.999998	.99999	0	0	-1.3*10 ⁻⁶	0.8431
				.01	.02	-0.005	0.8240
				.1	.2	-0.05	0.6266
α Cen AB	0.4519	,9971	.9850	0	0	.6818	2.1601
				.01	.02	.0624	2.1165
				.1	.2	.0100	1.6736
Kruger 60	0.3937	.999992	.99996	0	0	0.1062	1.1109
				.01	.02	0.1012	1.0934
				.1	.2	0.0562	0.9217
Xi Bootis	0.4231	.9988	.9998	0	0	0.0762	1.3729
				.01	.02	0.0713	1.3466
				.1	.2	0.0265	1.0813

Location of triangular equilibrium points

5.Conclusion

Considering the primaries of the elliptic restricted three body problem are oblate and radiating, we have seen that the locations of triangular points are perturbed due to including one or more one perturbing force of our model. These triangular points are stable within the domain $\mu c \in (0, 0.065)$ instead of Routhian classical value $\mu c = (0, 0.0384)$. The stability regions depend on the eccentricity of the primaries orbits, oblateness coefficients and the radiation parameters of the primaries. Appendix

$$\begin{split} D_{1,0} &= \frac{1}{30} \bigg[-30A_1 - 25A_1A_2 + 69A_1^2 - \frac{3525}{12}A_1^2A_2 + \frac{2205}{12}A_1A_2^2 \bigg] \\ D_{1,1} &= \frac{1}{30} \bigg[-\frac{135}{2}A_1 + 15A_2 - \frac{305}{2}A_1A_2 - \frac{583}{8}A_1^2 + \frac{25}{2}A_2^2 + \frac{78613}{120}A_1^2A_2 - \frac{2365}{12}A_1A_2^2 \bigg] \\ D_{1,2} &= \frac{1}{30} \bigg[-15A_1 + 30A_2 + \frac{115}{2}A_1A_2 + 28A_1^2 - 111A_2^2 - \frac{1295}{24}A_1^2A_2 - \frac{90543}{360}A_1A_2^2 \bigg] \\ D_{2,2} &= \frac{-15\sqrt{3}}{2} \bigg[\frac{45}{4}A_1 - \frac{81 + 45\sqrt{3}}{8}A_1A_2 + \frac{1620 + 353\sqrt{3}}{32}A_1^2 - \frac{433048 + 5355\sqrt{3}}{816}A_1^2A_2 - \frac{56 - 93\sqrt{3}}{32}A_1A_2^2 \bigg] \\ D_{2,3} &= \frac{-15\sqrt{3}}{2} \bigg[\frac{15 - \frac{84 - 105\sqrt{3}}{8}A_1 + \frac{160 - 15\sqrt{3}}{4}A_2 + \frac{9956 + 9843\sqrt{3}}{32}A_1A_2 - \frac{2082 + 1566\sqrt{3}}{16}A_1^2 \bigg] \\ D_{2,4} &= \frac{-15\sqrt{3}}{2} \bigg[\frac{45(1 + \sqrt{3})}{2}A_1 - \frac{60 - 45\sqrt{3}}{8}A_2 - \frac{780 - 1629\sqrt{3}}{16}A_1A_2 + \frac{69 + 630\sqrt{3}}{4}A_1^2 \bigg] \\ \end{split}$$

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6.References

6.1 Journal Article

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