"DETERMINATION OF RIJKE TUBE UPPER END IMPEDANCE USING EXPERIMENTAL VALUE OF LOWER END IMPEDANCE"

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ABSTRACT

Assuming zero impedance at the ends of Rijke tube combustor as used conventionally is found to be grossly in error. There is no self evident reason as to why this should be the case. In fact substantial acoustic energy is lost from the ends of the tube and this energy loss is not accounted for by the above boundary conditions. Loss of acoustic energy from the ends is proved by the author in [1] experimentally. Author had also determined impedance at lower end experimentally in [1]. Due to restriction on operating temperature, upper end impedance was not determined experimentally. In the present work author has developed a theoretical procedure to estimate upper end impedance using experimental value at the lower end. Results are in good agreement with expectation.

1. INTRODUCTION

The Rijke tube combustor [2] is of practical interest due to its potential for attaining high heat transfer and combustion efficiencies. A schematic diagram of Rijke combustor is shown in figure

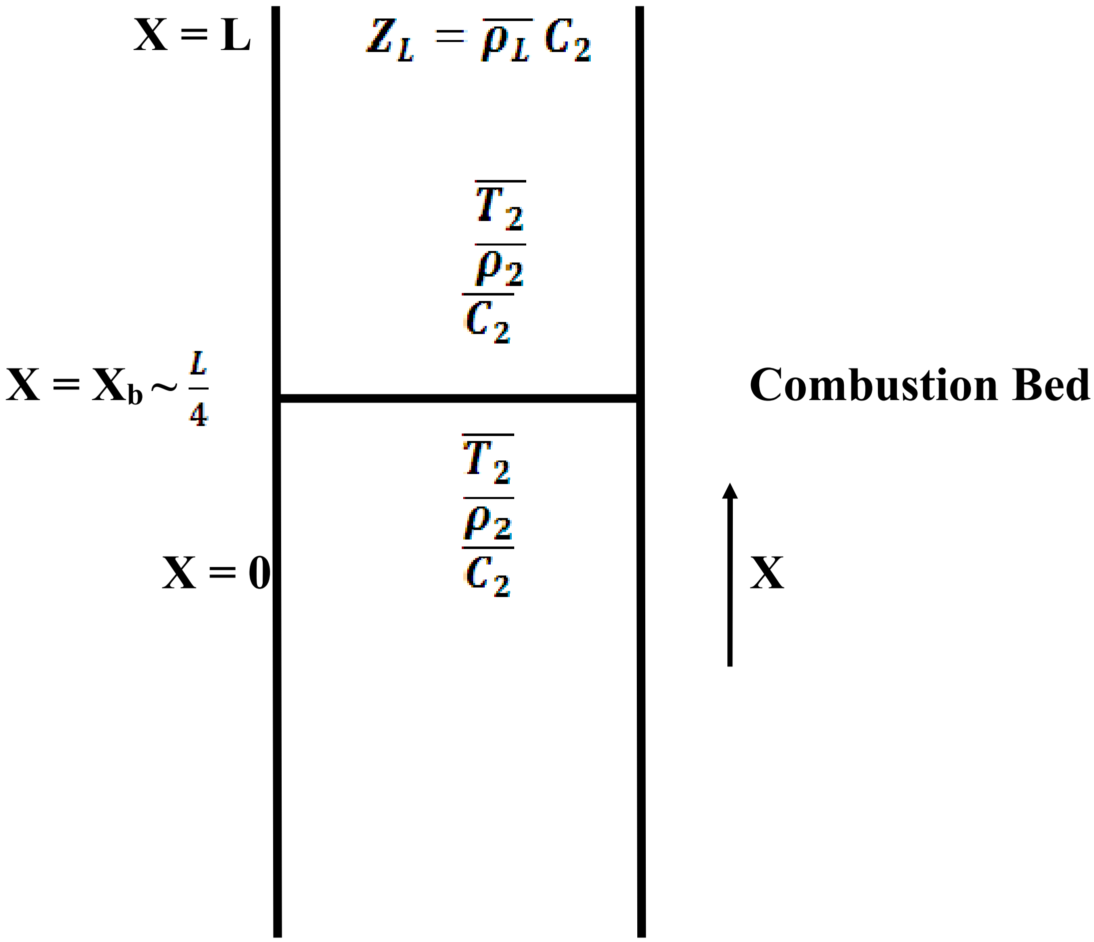

FIGURE : 1 - SCHEMATIC DIAGRAM OF RIJKE COMBUSTOR

1. results of [3] show that tube end impedance plays a vital role in determination of power output from the ends of the tube and from the combustion source itself and thus in combustion acoustic interaction in tube. However, the usual produce [4] assumes that the acoustic pressure p', thus impedance is equal to (or close to) zero at $x = 0$ and $x = L$. There is no self evident reason as to why this should be the case. In fact substantial acoustic energy is lost from the ends of the tube and this energy loss is not accounted for by the above boundary conditions. Clearly, the assumptions made above are inadequate.

The objective of the present work is to drive a simplified method for estimating upper end impedance using practical values of lower end impedance [1].

2. DERIVATION OF TUBE END IMPEDANCE (MAGNITUDE AND PHASE)

In order to derive the expression and value to upper end impedance the Rijke combustor is divided into two regions, separated by combustion bed. Steady state values are assigned as follows:

At $x = 0$, $z = z_0$

For $0 \le x \le x_b, c = c_1$ $\overline{\rho} = \rho_1$ $\overline{T} = \overline{T_1}$

For $x_b \le x \le L, c = c_2$ $\overline{\rho} = \rho_2$ $\overline{T} = \overline{T_2}$

At $x = L$, $z = z_L$

Thus the combustion bed is considered as a line of discontinuity in the steady state values of temperature, density and velocity of sound.

Under the assumption that the convective velocities in the combustor are small compared to the velocity of sound (this is almost always the case), isentropic solutions satisfying the tube end impedance boundary conditions can be obtained in the lower and upper regions of the tube. These are

for $0 \le x \le x_b$;

$$P_1 = A\left[e^{\left[\frac{i\omega x}{c_1}\right]} + \frac{1}{D_0}e^{-i\left[\theta_0 + \frac{\omega x}{c_1}\right]}\right]e^{-\omega t},\ V_1 \frac{A}{\overline{\rho_1 c_1}}\left[e^{\left[\frac{i\omega x}{c_1}\right]} - \frac{1}{D_0}e^{-i\left[\theta_0 + \frac{\omega x}{c_1}\right]}\right]e^{-\omega t} \quad \ldots(1)$$

where $D_0 e^{i\theta_0} = \dfrac{z_0 - \overline{\rho_1} c_1}{z_0 + \overline{\rho_1} c_1}$

and for $x_b \le x < L$

…(2)

$$P_2' = B\left[e^{\left[\frac{i\omega x}{c_2}\right]} + D_L e^{i\left[\frac{2wL}{c_2}+\theta_2-\frac{\omega x}{c_2}\right]}\right]e^{-\omega t} \quad V_2' = \frac{B}{\overline{\rho_2} c_2}\left[e^{\left[\frac{i\omega x}{c_2}\right]} - D_L e^{i\left[\frac{2\omega L}{c_2}+\theta_2-\frac{\omega x}{c_2}\right]}\right]e^{-\omega t}$$

where $D_L e^{i\theta_L} = \dfrac{z_L - \overline{\rho_2} c_2}{z_L + \overline{\rho}_2 c_2}$

Because combustion source is a line of discontinuity, the following matching conditions will be applicable.

$$P_1'(x_b) = P_2'(x_b) \text{ and } V_1'(x_b) = V_2'(x_b)$$

where

$$P_1'(x) = A\left[e^{\frac{i\omega x}{c_1}} + \frac{1}{D_0} e^{-i\left[\theta_0 + \frac{\omega x}{c_1}\right]}\right]e^{-i\omega t}$$

$$V_1'(x) = \frac{A}{\rho_1 c_1}\left[e^{\frac{i\omega x}{c_1}} - \frac{1}{D_0} e^{-i\left(\theta_0 + -\frac{\omega x}{c_1}\right)}\right]e^{-i\omega t}$$

where, $D_0 e^{i\theta_0} = \dfrac{z_0 - \rho_1 c_1}{z_0 + \rho_1 c_1}$

$$P_2'(x) = B\left[e^{\frac{i\omega x}{c_2}} + D_L e^{i\left(\frac{2\omega L}{c_2}+\theta_L-\frac{\omega x}{c_2}\right)}\right]e^{-i\omega t}$$

$$V_2'(x) = \frac{B}{\rho_2 c_2}\left[e^{\frac{i\omega x}{c_2}} - D_L e^{\left(i\frac{2\omega L}{c_2}+\theta_L-\frac{\omega x}{c_2}\right)}\right]e^{-i\omega t}$$

where, $D_L e^{i\theta_L} = \dfrac{z_L - \rho_2 c_2}{z_L + \rho_2 c_2}$

using matching conditions, $p_2'(x_b) = p_1'(x_b)$

$$B\left[e^{\frac{i\omega x_b}{c_2}}+D_L e^{i\left(\frac{2\omega L}{c_2}+\theta_L-\frac{\omega x_b}{c_2}\right)}\right]=A\left[e^{\frac{i\omega x_b}{c_1}}+\frac{1}{D_0}e^{-i\left(\theta_0+-\frac{\omega x_b}{c_1}\right)}\right]$$

and $v_2'(x_b)=v_1'(x_b)$

$$\rho_1 c_1 B\left[e^{\frac{i\omega x_b}{c_2}}-D_L e^{i\left(\frac{2\omega L}{c_2}+\theta_L-\frac{\omega x_b}{c_2}\right)}\right]=\rho_2 c_2 A\left[e^{\frac{i\omega x_b}{c_1}}-\frac{1}{D_0}e^{-i\left(\theta_0+\frac{\omega x_b}{c_1}\right)}\right] \quad \ldots(3)$$

solving the above equations,

$$A=\frac{B}{2\rho_2 c_2}(\rho_1 c_1+\rho_2 c_2)e^{\left(\frac{i\omega x_b}{c_2}-\frac{\omega x_b}{c_1}\right)}+D_L(\rho_2 c_2-\rho_1 c_1)e^{-i\left(\frac{2\omega L}{c_2}+\theta_L-\frac{\omega x_b}{c_2}-\frac{\omega x_b}{c_1}\right)}$$

Put it in above equation of $P_1'(x_b)=P_2'(x_b)$

$$(\rho_2 c_2-\rho_1 c_1)e^{\frac{i\omega x_b}{c_2}}+(\rho_1 c_1+\rho_2 c_2)D_L e^{i\left(\frac{2\omega L}{c_2}+\theta_L-\frac{\omega x_b}{c_2}\right)}$$

$$=\frac{(\rho_1 c_1+\rho_2 c_2)}{D_0}e^{i\left(\frac{\omega x_b}{c_2}-\theta_0-\frac{2\omega x_b}{c_1}\right)}+\frac{D_L}{D_0}(\rho_2 c_2-\rho_1 c_1)e^{i\left(\frac{2\omega L}{c_2}+\theta_L-\theta_0-\frac{\omega x_b}{c_2}-\frac{2\omega x_b}{c_1}\right)} \quad \ldots(4)$$

Equating real parts of equation (4) gives.

$$D_L=\frac{\left(\frac{\rho_1 c_1+\rho_2 c_2}{D_0}\right)\cos\left(\frac{\omega x_b}{c_2}-\theta_0-\frac{2\omega x_b}{c_1}\right)-(\rho_1 c_1+\rho_2 c_2)\cos\frac{\omega x_b}{c_2}}{(\rho_1 c_1+\rho_2 c_2)\cos\left(\frac{2\omega L}{c_2}+\theta_L-\frac{\omega x_b}{c_2}\right)-\frac{1}{D_0}(\rho_2 c_2-\rho_1 c_1)\cos\left(\frac{2\omega L}{c_2}+\theta_L-\theta_0-\frac{\omega x_b}{c_2}-\frac{2\omega x_b}{c_1}\right)} \quad \ldots(5)$$

Equating imaginary parts of equation (4) gives.

$$(\rho_2 c_2+\rho_1 c_1)\sin\frac{\omega x_b}{c_2}-\left(\frac{\rho_1 c_1+\rho_2 c_2}{D_0}\right)\sin\left(\frac{\omega x_b}{c_2}-\theta_0-\frac{2\omega x_b}{c_1}\right)$$

$$+D_L\left[(\rho_1 c_1+\rho_2 c_2)\sin\left(\frac{2\omega L}{c_2}+\theta_L-\frac{\omega x_b}{c_2}\right)-\frac{(\rho_2 c_2-\rho_1 c_1)}{D_0}\sin\left(\frac{2\omega L}{c_2}+\theta_L-\theta_0-\frac{\omega x_b}{c_2}-\frac{2\omega x_b}{c_1}\right)\right]=0$$

Now substituting value of D_L in equation from equation (5) gives the below equation

$$\left[(\rho_2 c_2)^2-(\rho_1 c_1)^2+\frac{1}{D_0^2}\times\left((\rho_2 c_2)^2-(\rho_1 c_1)^2\right)-2\times\frac{\left[(\rho_2 c_2)^2-(\rho_1 c_1)^2\right]}{D_0}\cos\left(\frac{2\omega \mathrm{x}_\mathrm{b}}{\mathrm{c}_1}+\theta_0\right)\right]$$

$$\sin\left(\frac{2\omega x_b}{c_2}-\frac{2\omega L}{c_2}-\theta_L\right)+\frac{4(\rho_2 c_2)(\rho_1 c_1)}{D_0}\cos\left(\frac{2\omega x_b}{c_2}-\frac{2\omega L}{c_2}-\theta_L\right)\sin\left(\frac{2\omega x_b}{c_2}+\theta_0\right)=0 \quad \ldots(6)$$

3. MATHEMATICAL ANALYSIS

The average temperature above the combustor bed = 1000K

The average temperature below the combustor bed = 600K

$$\therefore \rho_1=\frac{P}{RT_1}=\frac{1.01325\times 10^5}{287\times 600}=0.5884\,\text{kg/m}^3$$

$$C_1=\sqrt{1.4RT_1}=\sqrt{104\times 287\times 600}=491\,\text{m/s}$$

$$\rho_2=\frac{P}{RT_2}=\frac{1.01325\times 10^5}{287\times 1000}=0.353\,\text{kg/m}^3$$

$$c_2=\sqrt{1.4\times 287\times 1000}=633.87\text{ m/s}$$

Now from relation $D_0\cdot e^{i\theta_0}=\dfrac{Z_0-\rho_1 c_1}{Z_0+\rho_1 c_1}$

From relation, $Z_0=R+iE=-139.08+297.78i$,Using experimental value

from [5].

$$\therefore D_0\cdot e^{i\theta_0}=\frac{-139.08+297.78i-289}{-139.08+297.78i+289}$$

$$D_0\cdot e^{i\theta_0}=\frac{-428+298i}{149+298i}\times\frac{149-298i}{149-298i}$$

$$D_0\cdot e^{i\theta_0}=\frac{25182+171648i}{111005}=0.2268+1.55i$$

$$\therefore D_0 \cdot \cos\theta_0 + iD_0 \sin\theta_0 = 0.2268 + i1.55$$

$$D_0 = \sqrt{(0.2268)^2 + (1.55)^2} = 1.57$$

$$Tan\,\theta_0 = \frac{1.55}{0.2268} \Rightarrow \theta_0 = 81.67^0$$

Putting $\rho_1 = 0.5884,\ \rho_2 = 0.353, D_0 = 1.57, \theta_0 = 81.67^0,\ x_b = 0.69,\ \text{n} = 63,$

$c_1 = 491,\ c_2 = 634$ and $\text{L} = 2.75\ \text{m.}$ in equation (6)

$$\left[(0.353\times 634)^2 - (0.5884\times 491)^2 + \frac{1}{(1.57)^2}\times\left((0.353\times 634)^2 - (0.5884\times 491)^2\right)\right.$$

$$\left. - \frac{2}{1.57}\left((0.353\times 634)^2 + (0.5884\times 491)^2\right)\cos\left(\frac{4\times 22\times 63\times 0.69}{7\times 491} + 81.67^0\right)\right]\times$$

$$\sin\left(\frac{4\times 22\times 63\times 0.69}{7\times 634} - \frac{4\times 22\times 63\times 2.75}{7\times 634} - \theta_L\right) + \frac{4\times 0.5884\times 491\times 0.353\times 634}{1.57}$$

$$\cos\left(\frac{4\times 22\times 63\times 0.69}{7\times 634} - \frac{4\times 22\times 63\times 2.75}{7\times 634} - \theta_L\right)\sin\left(\frac{4\times 22\times 63\times 0.69}{7\times 634} + 81.67\right) = 0$$

$$[-46784 - 169470\times(-0.83)]\sin(-147 - \theta_L) + 164620\cos(\text{-}147\text{ - }\theta_{\text{L}})(0.75) = 0$$

$$93876\sin(\text{-}147\text{ - }\theta_{\text{L}}) = -123465\ \cos\ (\text{-}147\text{ - }\theta_{\text{L}})$$

$$Tan\,(\text{-}147\text{ - }\theta_{\text{L}}) = -1.3152$$

$$\text{-}147\text{ - }\theta_{\text{L}} = -52.7$$

$$\theta_{\text{L}} = -94.3^0$$

From equation (5)

$$\frac{(0.5884\times 491 + 0.353\times 634)}{1.57}\cos\left(\frac{2\times 22\times 63\times 0.69}{7\times 634} - 81.67 - \frac{4\times 22\times 63\times 0.69}{7\times 491}\right)$$

$$D_L = \frac{-\left(0.353\times 634 - 0.5884\times 491\right)\cos\left(\frac{2\times 22\times 63\times 0.69}{7\times 491}\right)}{\left(0.5884\times 491 + 0.353\times 634\right)\cos\left(\frac{4\times 22\times 63\times 2.75}{7\times 634} - 94.3 - \frac{2\times 22\times 63\times 0.69}{7\times 634}\right)}$$

$$-\frac{1}{1.57}\left(0.353\times 634 - 0.5884 - 491\right)\cos\left(\frac{2\times 22\times 63\times 2.75}{7\times 634} + 94.3 - 81.67 - \frac{2\times 22\times 63\times 0.69}{7\times 634}\right.$$

$$\left. - \frac{4\times 22\times 63\times 0.69}{7\times 491}\right)$$

$$D_L = \frac{326.75\cos(-82-39)(-65)\cos 32}{513\cos 80 + 4.4\cos(-65)}$$

$$= \frac{-168+55}{89+17} = \frac{-113}{141} = -0.8$$

4. RESULTS:- DETAILED ANALYSIS GIVES

$$\theta_0 = 81.67^0$$

$$D_0 = 1.57$$

$$\theta_{\mathrm{L}} = -94^0$$

$$D_{\mathrm{L}} = -0.8$$

$$Z_{\mathrm{L}} = -225 - 7i$$

$$\therefore \left|Z_L\right| = 225$$

or $Z_{\mathrm{L}} = 30 - 236i$

$$\left|Z_L\right| = 237$$

These values are in good agreement with the optimum values used in [3].

5. CONCLUSIONS AND FUTURE SCOPE

a. The use of a zero impedance boundary condition at the open ends of Rijke tube combustors is not appropriate. A more realistic values is found to be $|z| = \overline{\rho}c$

b. By analysis it is evident that acoustic energy is lost from the ends. Loss of acoustic energy at the ends is accounted by realistic value of $|z|$.

***Nomenclature**

A,B = Arbitrary constants

C = Velocity of sound

D = constant, characterizing tube and impedance.

e = Base of natural logarithms

i = $\sqrt{-1}$

L = Tube length

P = Pressure

T = Time

V = Velocity

X = Length co-ordination

Z = Acoustic impedance

ρ = Density

ω = Angular frequency

θ = Phase, characterizing tube end impedance

***SUPER SCRIPTS AND SUBSCRIPTS**

$(\ )'$ = Acoustic component

$(\ )_0$ = at $x = 0$

$(\ )_l$ = in the region $0 \le x \le x_b$

$(\bar{\ })$ = steady state component.

$(\ )_b$ = at the combustion bed, i.e., $x = x_b$

$(\ )_2$ = in the region $x_b \le x \le L$

(L) = at $x = L$

5. REFERENCES

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