Study of Supply Chain Inventory Model with Time Dependent Demand under Partial Backlogging

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Abstract

In the present paper, we studied about deteriorating items with the policy over a fixed planning period for a deteriorating item which is downgrading of the effectiveness or physical characteristics of a substance due to faulty packaging or abnormal storage conditions. The model is systematically by minimizing the total inventory cost and can be modify the total inventory cost for the business management where both holding cost and deterioration rate are constant.

Keywords: Production, Inventory, Deterioration, Partial Backlogging.

2. INTRODUCTION

Inventory is essential to provide flexibility in operating a system or organization. An inventory can be classified into raw material inventory, work-in-process inventory and finished goods inventory. The raw material removes dependency between suppliers and plants. The work-in-process inventory removes dependency between various machines of a product line. The finished goods inventory removes dependency plants and its customers or market. An inventory consists of usable but idle resources such as men, machines, materials or money. When the resource involved is a material, the inventory is also called ‘stock’. Inventory management is the management of inventory and stock. **Silver and Meal** [1] presented the inventory replenishment policy over a fixed planning period for a deteriorating item having a deterministic demand pattern with a linear trend and shortages. The model is solved analytically by minimizing the inventory cost. **Donaldson** [2] declare the classical no-shortage inventory policy is examined for the case of a linear trend in demand. Using methods of calculus a computationally simple procedure for determining the optimal times for replenishment of inventory is established. Table extracts for the functions required are included. **Silver [3]**relaxes the restrictive assumption of constant demand rate that is often used in the inventory economic order quantity model and also proposes a simple fixed-horizon algorithm for the inventory replenishment problem with a linear trend in demand. The algorithm is based on an iterative numerical procedure that generates the optimal replenishment schedule for both growing and declining markets. Two numerical examples from the literature are included to illustrate the algorithm. **Ritchie [4]** represents the E.O.Q. is derived for constant, continuous demand. For linear increasing demand, Donaldson's analytical solution considers demand bounded by a time horizon H. Although this is mathematically convenient for obtaining a solution, it complicates the calculation of the optimal replenishment policy. Extending the time horizon so that it no longer influences the replenishment times simplifies the calculation of the optimal policy, which is then equivalent to the E.O.Q. calculation for constant demand.

Mitra [5] purposed the inventory model over a period of fixed planning for a deteriorating item having a selling price demand rate in which shortages are allowed and are partially backlogged. The results are verified with a numerical example. **Harris[6]**a time dependent inventory model is developed on the basis of constant production rate and market demands which are exponentially decreasing. It advances in quest of total average optimum cost considering those products which have finite shelf-life. The model also considers the small amount of decay. Without having any sort of backlogs, production starts. **Wilson [7]** a total optimal cost of an inventory model with exponential declining demand and constant deterioration is considered. The time-varying holding cost is a linear function of time. Shortages are not allowed. The items (like food grains, fashion apparels and electronic equipments) have fixed shelf-life which decreases with time during the end of the season. A numerical example is presented to demonstrate the model and the sensitivity analysis of various parameters is carried out. **Whitin, T. M. [8]** describe the inventory theory and the inventory replenishment policy over a fixed planning period for a deteriorating item having a deterministic demand pattern with a linear trend and shortages. The model is solved analytically by minimizing the total inventory cost. **Dave and Patel [9]** represented An EOQ model is reconsidered here in which the demand rate is changing linearly with time and the deterioration is assumed to be a constant fraction of the on hand inventory. The planning horizon is finite and known and the replenishment periods are assumed to be constant. **Ghare and Schrader [10]** presented A production inventory model has been developed in this paper, basing on constant production rate and market demand, which varies time to time. Seeing the demand pattern the proposed model has been formulated in a power pattern which can be expressed in a linear or exponential form. **Covert and Philip [11]** describe Uncertainty is the natural phenomena for any type of business transaction. Deterioration is not always constant during market period. This paper considers an inventory model for exponential demand rate and waybill distribution deterioration. Shortages are allowed and linearly time dependent. **Shah [12]** describe an attempt has been made to develop a production lot size model which incorporates an unfilled-order backlog for an inventory system with exponential decaying items. Approximate expressions are obtained for the optimum production lot size, the production cycle time and the total cycle time. **Elsayed and Teresi [13]** declare two economic order quantity models (1 and II) for inventory items with deterioration rates are developed. In Model I, the demand is deterministic in nature, production rate is finite, and shortages are allowed. In Model II, the demand is given by a random variable having a normal distribution, the deterioration rate is given by a two-parameter Weibull distribution, and shortages are allowed. **Rafaet, F. F.; Wolfe, P. M.; Kldin, H. K. [14]**develop a deterministic inventory model for single deteriorating items with two separate storage facilities due to limited capacity of the existing storage. The demand rate of the item is dependent on time, selling price of an item, and the frequency of advertisement in the popular electronic and print media and also through the sales representatives. **Heng, K. J., Labban, J., Linn, R. J**.**[15]** represents an inventory model for deteriorating and repairable item developed with linear demand. It extends a lot size inventory model with inventory-level dependent demand. Shortages are partially backordered and defective product can be repaired. Optimal lot size with minimum total cost is derived using Taylor and calculus analysis.

Mandal, B. [16] presented an inventory model is considered in which it is depleted not only by demand, but also by deterioration. The Waybill distribution, which is capable of representing constant, increasing and decreasing rates of deterioration, is used to represent the distribution of the time to deterioration. **Mishra, V. K.; Singh, L. S.[17]** considered a deterministic inventory model with time-dependent demand and time-varying holding cost where deterioration is time proportional. The model considered here allows for shortages, and the demand is partially backlogged. The model is solved analytically by minimizing the total inventory cost. The result is illustrated with numerical example for the model. **Mishra, V. K.; Singh, L. S. [18]** declares a differential equation inventory model that incorporates partial backlogging and deterioration. Holding cost and demand rate are time dependent. Shortages are allowed and assumed to be partially backlogged. **Hung, K. C.[19] e**xtend their inventory model from ramp type demand rate and Weibull deterioration rate to arbitrary demand rate and arbitrary deterioration rate in the consideration of partial backorder. We demonstrate that the optimal solution is actually independent of demand. That is, for a finite time horizon, any attempt at tackling targeted inventory models under ramp type or any other types of the demand becomes redundant. Our analytical approach dramatically simplifies the solution procedure. **Mishra, V. K.; Singh, L.; Kumar, R. [20]** purposed a total optimal cost of an inventory model with exponential declining demand and constant deterioration is considered. The time-varying holding cost is a linear function of time. Shortages are not allowed. The items (like food grains, fashion apparels and electronic equipments) have fixed shelf-life which decreases with time during the end of the season. Pandey and Pandey [21] have developed an Inventory Model for Deteriorating Items considering two level storage with uniform demand and shortage under Inflation and completely backlogged. Pandey, H. and Pandey, A. [22] have developed an optimum inventory policy for exponentially deteriorating items, considering multi variate Consumption Rate with Partial Backlogging. Pandey et al.[23] have studied an EOQ Model with Ramp Type of Demand. Kumar et al [24] developed an Integrated Model with Variable Production and Demand Rate under Inflation. Pandey et al.[25] studied a Study of Production Inventory Policy with Stock Dependent Demand Rate. Pandey, A. [26] studied an EPQ Inventory Model for Deteriorating Items Considering Stock Dependent Demand and Time Varying Holding Cost. Again Pandey et al.[27] investigated an EOQ MODEL WITH QUANTITY INCENTIVE STRATEGY FOR DETERIORATING ITEMS AND PARTIAL BACKLOGGING.

3. Model Formulazion and Solution

3.1 NOTATION AND ASSUMPTION

The fundamental assumption and notation used in this paper are given as below:

a) The demand rate is time dependent and linear, i.e. $D(t) = a + bt;\ a, b > 0$ and are constant.

b) The replenishment arte is infinite, thus replenishment is instantaneous.

c) I(t) is the level of inventory at time t, $0 \leq t \leq T$.

d) T is the length of the cycle.

e) θ is the constant deteriorating rate, 10<<θ.

f) t_1 is the time when the inventory level reaches zero.

g) $t_1{}^*$ is the optimal point.

h) Q is the ordering quantity per cycle.

i) A_0 is the fixed ordering cost per order.

j) C_1 is the cost of each deteriorated item.

k) C_2 is the inventory holding cost per unit per unit of time.

l) $C_3$is the shortage cost per unit per unit of time.

m) S is the maximum inventory level for the ordering cycle, such that S=I(0).

n) $C_1(t_1)$ is the average total cost per unit time under the condition $t_1 \leq T$.

3.2 MATHEMATICAL FORMULATION

Here we consider the deteriorating inventory model with linearly time dependent demand rate. Replenishment occurs at time t=0 when the inventory level attains its maximum. From t=0 to t_1, the inventory level reduces due to demand and deterioration. At t_1, the inventory level achieves zero, then shortage is allowed to occur during the time interval (t_1, T) is completely backlogged. The total number of backlogged items is replaced by the next replenishment. According to the notations and assumptions mentioned above, the behavior of inventory system at any time can be described by the following differential equations:

$$\frac{\mathrm{dI(t)}}{\mathrm{dt}} + \theta I(t) == -a - bt, \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI(t)}{dt} == -a - bt, \qquad \mathrm{t_1} \le \mathrm{t} \le \mathrm{T} \tag{2}$$

Solution of equation number (1) and (2) with boundary condition I(0)=S, I(t_1)=0 are as follows.

$$\mathrm{I(t)}= \frac{b}{\theta^2} - \frac{be^{-t\theta+\mathrm{t1}\theta}}{\theta^2} - \frac{a}{\theta} + \frac{ae^{-t\theta+\mathrm{t1}\theta}}{\theta} - \frac{bt}{\theta} + \frac{be^{-t\theta+\mathrm{t1}\theta}\mathrm{t1}}{\theta}, \quad 0 \le t \le t_1 \tag{3}$$

$$\mathrm{I(t)}= -aT - \frac{bT^2}{2} + a\mathrm{t}_1 + \frac{b\mathrm{t1}^2}{2}, \qquad t_1 \le t \le T \tag{4}$$

The beginning inventory level can be computed as

$$S = I(0) = \frac{b}{\theta^2} - \frac{be^{\mathrm{t1}\theta}}{\theta^2} - \frac{a}{\theta} + \frac{ae^{\mathrm{t1}\theta}}{\theta} + \frac{be^{\mathrm{t1}\theta}\mathrm{t1}}{\theta} \tag{5}$$

The total number of items which perish in the interval [0,t_1], sayD_t, is

$$D_T = S - \int_0^{t_1}(a+bt)dt$$

$$-a\mathrm{t}_1 - \frac{b\mathrm{t1}^2}{2} + \frac{b}{\theta^2} - \frac{be^{\mathrm{t1}\theta}}{\theta^2} - \frac{a}{\theta} + \frac{ae^{\mathrm{t1}\theta}}{\theta} + \frac{be^{\mathrm{t1}\theta}\mathrm{t1}}{\theta} \tag{6}$$

The total number of items which perish in the interval [0,t_1], sayH_T, is

$$H_T = \int_0^{t_1} I(t)dt$$

$$= \int_0^{t_1}\left[\frac{b}{\theta^2} - \frac{be^{-t\theta+\mathrm{t1}\theta}}{\theta^2} - \frac{a}{\theta} + \frac{ae^{-t\theta+\mathrm{t1}\theta}}{\theta} - \frac{bt}{\theta} + \frac{be^{-t\theta+\mathrm{t1}\theta}\mathrm{t1}}{\theta}\right]\mathrm{dt}$$

$$= \frac{b}{\theta^3} - \frac{be^{\mathrm{t1}\theta}}{\theta^3} - \frac{a}{\theta^2} + \frac{ae^{\mathrm{t1}\theta}}{\theta^2} + \frac{be^{\mathrm{t1}\theta}\mathrm{t1}}{\theta^2} - \frac{a\mathrm{t1}}{\theta} - \frac{b\mathrm{t1}^2}{2\theta} \tag{7}$$

The total shortage quantity during the interval $[t_1, T]$, say B_T, is

$B_T = -\int_{t_1}^{T} I(t)dt$

$= -\int_{t_1}^{T}\left[-aT - \frac{bT^2}{2} + at_1 + \frac{bt1^2}{2}\right]dt$

$= aT^2 + \frac{bT^3}{2} - 2aTt1 - \frac{1}{2}bT^2t1 + at1^2 - \frac{1}{2}bTt1^2 + \frac{bt1^3}{2}$ (8)

Then, the average total cost per unit time under the condition $t_1 \leq T$ can be given by

$c_1(t_1) = \frac{1}{T}[A_0 + C_1D_T + C_2H_T + C_3B_T]$ (9)

$= \frac{1}{T}\Big[A_0 + \left(-at_1 - \frac{bt1^2}{2} + \frac{b}{\theta^2} - \frac{be^{t1\theta}}{\theta^2} - \frac{a}{\theta} + \frac{ae^{t1\theta}}{\theta} + \frac{be^{t1\theta}t1}{\theta}\right)C_1 + \left(\frac{b}{\theta^3} - \frac{be^{t1\theta}}{\theta^3} - \frac{a}{\theta^2} + \frac{ae^{t1\theta}}{\theta^2} + \frac{be^{t1\theta}t1}{\theta^2} - \frac{at1}{\theta} - \frac{bt1^2}{2\theta}\right)C_2 + \left(aT^2 + \frac{bT^3}{2} - 2aTt1 - \frac{1}{2}bT^2t1 + at1^2 - \frac{1}{2}bTt1^2 + \frac{bt1^3}{2}\right)C_3\Big]$

The first order derivative of $C_1(t_1)$ with respect to $t_1 \leq T$ can be given by

$\frac{dC_1(t_1)}{dt_1} = \frac{1}{T}((-a + ae^{t1\theta} - bt1 + be^{t1\theta}t1)C_1 + (-\frac{a}{\theta} + \frac{ae^{t1\theta}}{\theta} - \frac{bt1}{\theta} + \frac{be^{t1\theta}t1}{\theta})C_2 + (-2aT - \frac{bT^2}{2} + 2at1 - bTt1 + \frac{3bt1^2}{2})C_3)$ (10)

The necessary condition for $C_1(t_1)$ in (9) to be minimized is

$$\frac{dC_1(t_1)}{dt_1} = 0, \textit{that is}$$

$\frac{1}{T}\left[\left(-a + ae^{t1\theta} - bt1 + be^{t1\theta}t1\right)C_1 + \left(-\frac{a}{\theta} + \frac{ae^{t1\theta}}{\theta} - \frac{bt1}{\theta} + \frac{be^{t1\theta}t1}{\theta}\right)C_2 + \left(-2aT - \frac{bT^2}{2} + 2at1 - bTt1 + \frac{3bt1^2}{2}\right)C_3\right] = 0$ (11)

$\left[\left(-a + ae^{t1\theta} - bt1 + be^{t1\theta}t1\right)C_1 + \left(-\frac{a}{\theta} + \frac{ae^{t1\theta}}{\theta} - \frac{bt1}{\theta} + \frac{be^{t1\theta}t1}{\theta}\right)C_2 + \left(-2aT - \frac{bT^2}{2} + 2at1 - bTt1 + \frac{3bt1^2}{2}\right)C_3\right] = 0$

Let $g(t_1) = \left[\left(-a + ae^{t1\theta} - bt1 + be^{t1\theta}t1\right)C_1 + \left(-\frac{a}{\theta} + \frac{ae^{t1\theta}}{\theta} - \frac{bt1}{\theta} + \frac{be^{t1\theta}t1}{\theta}\right)C_2 + \left(-2aT - \frac{bT^2}{2} + 2at1 - bTt1 + \frac{3bt1^2}{2}\right)C_3\right]$

From equation (11) we will get the value of t_1 which minimized equation (9).

CONCLUSION

In the present study, the inventory model for deteriorating items with linear time dependent demand rate is developed. We proposed an inventory replenishment policy for this type of inventory model .The replenishment arte is infinite, thus replenishment is instantaneous. The study provides an interesting topic for the further study on considering of important inventory models for supply chain management.

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