A new way of solving Fuzzy Transportation Problem using Fuzzy New Method

Althada Ramesh Babu[1 & 2*], B. Rama Bhupal Reddy[3]

[1] Research Scholar, Department of Mathematics, JNTUA, Ananthapuramu, Andhra Pradesh, INDIA
[2] Associate Professor, Department of AS&H, Sasi Institute of Technology & Engineering, Tadepalligudem, W.G.Dt, Andhra Pradesh, Pincode 534101- INDIA.
[3] Professor, Department of Mathematics, KSRM College of Engineering, Kadapa, Andhra Pradesh, INDIA.
E-mail: [1] rameshbabu.althada@sasi.ac.in [2] reddybrb@gmail.com

Abstract

This paper aims at finding the easiest method of optimal solution of Fuzzy Transportation problem (FTP). In order to solve FTP, it needs to be converted into crisp problem since it contains membership function with corresponding intervals so that Robust Ranking technique is used. Later Initial Basic Feasible Solution (IBFS) is obtained using the two methods (Fuzzy Vogel Approximation Method & Fuzzy New Method. Finally, optimal solution is found using fuzzy stepping stone method [8]. The solution procedure is illustrated with a numerical example.

Keywords: Trapezoidal fuzzy membership function [9], Robust Ranking technique, Initial basic feasible solution, Fuzzy new method, Fuzzy Vogel approximation method, Fuzzy steeping stone method

1. Introduction

The transportation problem refers to a special case of linear programming problem. The basic transportation problem was developed by Hitchcock [1]. In Mathematics and Economics transportation theory is a name given to the study of optimal transportation and allocation of resources. Transportation problems deal with the distribution of single commodity from various sources of supply to various destinations of demand in such a manner that the total transportation cost is minimized. In order to solve a transportation problem the decision parameters such as availability, requirements and the unit cost of transportation the model must be fixed at crisp values. But in real life applications supply, demand and unit transportation cost may be uncertain due the several factors. These imprecise data may be represented by fuzzy numbers.

The idea of fuzzy set was introduced by L.A. Zadeh [2] in 1965. After this pioneering work many authors have studied fuzzy linear programming problem techniques, A method for solving a fuzzy transportation problem with Crisp function[3] which provides only crisp solution to the given problem. The fuzzy transportation problem can be solved by fuzzy linear programming techniques. But most of the existing techniques provide the crisp solution of the fuzzy transportation problem. Ranking method is used to change fuzzy numbers into crisp form. In this paper, an algorithm is proposed from the Fuzzy Vogel Approximation Method[4] & Fuzzy New Method[4,6&7].

2. Preliminaries

2.1 Basic definitions

Definition:1 A Fuzzy set A is defined as the set of ordered pairs (X, $\mu_A(x)$), where x is an element of the universe of discourse U and $\mu_A(x)$ is the membership function that attributes to each x ∈ U a real number ∈ [0, 1] describing the degree to which X belongs to the set.

Definition:2 A crisp set is a special case of Fuzzy set, in which the membership function takes only two values 0 and 1.

Definition:3 A fuzzy number $\bar{A}$ = (a, b, c, d) is said to be a trapezoidal fuzzy number if its membership function $\mu_{\bar{A}}(x)$ is given by,

$$\mu_{\bar{A}}(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ 1, \quad b \le x \le c \\ \frac{d-x}{d-c}, c \le x \le d \\ 0, \quad otherwise \end{cases}$$

Graphically, a trapezoidal fuzzy number can be represented (figure 2.1.1):

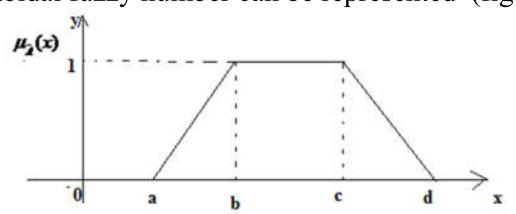

Figure 2.1.1. Trapezoidal fuzzy number.

2.2 Arithmetic operations

Let (a_1 ,a_2, a_3, a_4) and (b_1, b_2, b_3, b_4) be two Trapezoidal fuzzy numbers. Then

(i) (a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4) = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)

(ii) (a_1, a_2, a_3, a_4) - (b_1, b_2, b_3, b_4) = (a_1-b_1, a_2-b_2, a_3-b_3, a_4-b_4)

(iii) k(a_1, a_2, a_3, a_4) = (ka_1, ka_2, ka_3, ka_4) for $k \geq 0$

(iv) k(a_1, a_2, a_3, a_4) = (ka_4, ka_3, ka_2, ka_1) for $k < 0$

3. Mathematical Formulation of Fuzzy Transportation Problem

A fuzzy Transportation Problem can be stated as Min z = $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ Subject to $\sum_{j=1}^{n} x_{ij} = a_i, i = 0,1,2,...m$, $\sum_{i=1}^{m} x_{ij} = b_j, j = 0,1,2,...n$, where $x_{ij} \geq 0, i = 1,2,3...m, \ j = 1,2,3...n$ where i = 0,1,2,…,m in which the transportation cost c_{ij}, supply a_i, and b_j quantities are fuzzy quantities. The necessary and sufficient condition for the fuzzy linear programming is given as $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ (RIM conditions or balancing transportation problem)

4. Computational Procedure

4.1 **Robust Ranking Technique**:

Robust Ranking technique [8] which satisfies compensation, linearity, and additive properties and provides results which are consist human intuition. If ã is a fuzzy number then the Robust Ranking is defined by R(ã) = $\int_0^1 0.5(a_\alpha^L, a_\alpha^U)d\alpha$ where (a_α^L, a_α^U) is the α level cut of the fuzzy number ã and

$(a_\alpha^L, a_\alpha^U) = \{((\alpha(b-a)+a)+(d-(d-c)\alpha)\}$ In this paper it is attempted to use this method for ranking the objective values. The Robust ranking index R(ã) gives the representative value of fuzzy number ã

4.2 Finding Initial Basic Feasible Solution:

Here we are using two methods to find IBFS. So let us discuss the procedure of two methods

4.2.1 The Computation procedure for **Fuzzy Vogel's Approximation Method (FVAM)**

The steps for finding IBFS using FVAM are as follow:

Step:1. Take the first row and choose its smallest entry and subtract this from next smallest entry, and write in front of the row on the right. This is the fuzzy penalty for first row. Similarly compute fuzzy penalties for all the columns and write them in the bottom of the Fuzzy Transportation Problem (FTP) below corresponding columns.

Step:2. Select the highest fuzzy penalty and observe the row or column for which this corresponds. Determine the smallest fuzzy cost in the selected row or column. Let it be c_{ij}.

Find x_{ij} = minimum (a_i , b_j)

There may arise the following three cases:

Case(i) If minimum $(a_i , b_j) = a_i$ then allocate $x_{ij} = a_i$ in the (i , j)th cell of mxn FTP. Ignore the i^{th} row to obtain a new FTP of order (m-1)xn. Replace b_j by $b_j - a_i$ in obtained FTP. Go to Step 3.

Case(ii) If minimum $(a_i , b_j) = b_j$ then allocate $x_{ij} = b_j$ in the (i , j)th cell of mxn FTP. Ignore the j^{th} column to obtain a new FTP of order mx(n-1). Replace a_i by $a_i - b_j$ in obtained FTP. Go to Step 3

Case(iii) If minimum $a_i = b_j$ then either follow Case (i) or Case (ii) but not both, simultaneously. Go to step2

Step:3. Calculate fresh penalties for the obtained FTP as in Step 1. Repeat Step 2, until the FTP is reduced into FTP of order 1x1.

Step:4. Allocate all x_{ij} in the (i,j)th cell of the given FTP.

Step:5. The IFBFS solution and initial fuzzy transportation cost are x_{ij} and $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}$, respectively. This process gives the initial basic feasible solution and we can optimize using Fuzzy stepping stone method if it is non-degenerate.

4.2.2 The Computation procedure of **Fuzzy New Method (FNM):**

To find the fuzzy initial basic feasible solution (FIBFS) of a fuzzy transportation problem the following algorithm is proposed:

Step 1 : Find the Fuzzy penalty cost ie. fuzzy difference between the maximum fuzzy cost and minimum fuzzy cost in each row and column.

Step 2 : Choose a row or column with maximum penalty by ranking method. If maximum penalty is more than one choose any one arbitrarily.

Step 3 : From the selected row or column choose a fuzzy minimum cost and allocate as much as possible in that cell depending on supply and demand.

Step 4 : Delete the row or column which is fully exhausted. Repeat the process till, all the RIM R are satisfied.

4.3 Finding optimality of Fuzzy transportation problem by Fuzzy stepping stone method

4.3.1 To find the fuzzy optimal solution of a fuzzy transportation problem then the algorithm is proposed:

Step 1: Find an initial basic feasible solution using any one of the three methods FVAM & FNM

Step 2: a) Draw a closed path from the unoccupied cell. The right angle turn in this path is allowed only at occupied cells and at the original unoccupied cell. Mark (+/-)sign

alternatively at each corner, starting from the original unoccupied cell.
b) Add the transportation costs of each cell traced in the closed path. This is called net cost.
c) Repeat this for all other unoccupied cells.

Step 3: a) If all the net cost change are ≥ 0, an optimal solution has been reached. Now stop this procedure.
b) If not then select the unoccupied cell having the highest negative net cost change and draw a closed path.

Step 4: a) Select minimum allocated value among all negative position (-) on closed path
b) Assign this value to selected unoccupied cell (So unoccupied cell becomes occupied).
c) Add this value to the other occupied cells masssrked with (+) sign.
d) Subtract this value to the other occupied cells marked with (-) sign.

Step 5: Repeat step-2 to step-4 until optimal solution is obtained. This procedure stops when all net cost change ≥0 for unoccupied cells

5. Numerical Example

Problem: To illustrate the new method let us consider a fuzzy transportation

Table 5.1

	D1	D2	D3	D4	Availability
O1	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(4,6,7,9)
O2	(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
O3	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,15)
Requirement	(5,7,8,10)	(-1,5,6,7)	(1,3,4,6)	(1,2,3,4)	(9,17,21,27)

Solution: Let us find the ranks of each cell using Robust ranking technique:

R(ã) = $\int_0^1 0.5(a_\alpha^L, a_\alpha^U)d\alpha$ where (a_α^L, a_α^U) is the α level cut of the fuzzy number ã and

$(a_\alpha^L, a_\alpha^U) = \{((\alpha(b-a)+a)+(d-(d-c)\alpha)\}$

So rank of a_{11} = $\bar{a}(1,2,3,4) = \int (0.5)((2-1)\alpha + 1 + 4 - (4-3)\alpha)d\alpha = 2.5$

Table 5.2

	D1	D2	D3	D4	Av
O1	(1,2,3,4:2.5)	(1,3,4,6:3.5)	(9,11,12,14:11.5)	(5,7,8,11:7.5)	(4,6,7,9:6.5)
O2	(0,1,2,4:1.5)	(-1,0,1,2:0.5)	(5,6,7,8:6.5)	(0,1,2,3:1.5)	(0,1,2,3:1.5)
O3	(3,5,6,8:5.5)	(5,8,9,12:8.5)	(12,15,16,19:15.5)	(7,9,10,12::9.5)	(5,10,12,15:11)
Req	(5,7,8,10:7.5)	(-1,5,6,7:5.5)	(1,3,4,6:3.5)	(1,2,3,4:2.5)	(9,17,21,27:19)

In the above table (a, ,b, c, d:e) e indicates rank of the cell value.
So the crisp problem be based on ranking shown below
Note: Req: requirement, Av: Availability

Table 5.3

	D1	D2	D3	D4	Av
O1	2.5	3.5	11.5	7.5	6.5
O2	1.5	0.5	6.5	1.5	1.5
O3	5.5	8.5	15.5	9.5	11
Req	7.5	5.5	3.5	2.5	19

Step:-II: finding IBPS (using method-I by Fuzzy Vogel approximation method)

Table 5.4

	D1	D2	D3	D4	Av	Row Penality
O1	2.5	3.5	11.5	7.5	6.5	1
O2	1.5	0.5	6.5	1.5(1.5)	~~1.5~~	1
O3	5.5	8.5	15.5	9.5	11	3
Req	7.5	5.5	3.5	~~2.5~~ 1		
Column Penality	1	3	5	6 **		
Note: ** Maximum penality						

Above process continue…

Table 5.5 Final table for IBPS

	D1	D2	D3	D4	Av
O1	2.5(6.5)	3.5	11.5	7.5	6.5
O2	1.5	0.5	6.5	1.5(1.5)	1.5
O3	5.5(1)	8.5(5.5)	15.5(3.5)	9.5(1)	11
Req	7.5	5.5	3.5	2.5	

The IBFS be Min Z = (2.5*6.5) + (1.5*1.5) + (5.5*1) + (8.5*5.5) + (15.5*3.5) + (9.5*1) = 134.5

Method ii: Fuzzy New Method (FNM):

Table 5.6

	D1	D2	D3	D4	Av	Row Penalty
O1	2.5	3.5	11.5	7.5	6.5	9
O2	1.5	0.5	6.5	1.5	1.5	6
O3	5.5(7.5)	8.5	15.5	9.5	~~11~~ 3.5	10**
Req	~~7.5~~	5.5	3.5	2.5		
Column Penalty	4	8	9	8		
Note: ** Maximum penalty						

Above process continue…

Table 5.7 Final table for IBPS

	D1	D2	D3	D4	Av
O1	2.5	3.5(5.5)	11.5(1)	7.5	6.5
O2	1.5	0.5	6.5(1.5)	1.5	1.5
O3	5.5(7.5)	8.5	15.5(1)	9.5(2.5)	11
Req	7.5	5.5	3.5	2.5	

Soln be z = (3.5*5.5) + (11.5*1) +(6.5*1.5) + (5.5*7.5)+ (15.5*1) + (9.5*2.5)=121

Comparison table for IBFS: Table 5.8

Method	No.of iterations	IBFS
FVAM	8	134.5
FNM	6	**121**

Step:-III: finding optimal solution using Fuzzy Stepping Stone method

Case-i: For **Table 5.5,** let us find optimal solution using stepping stone method

Step-I Checking for degeneracy of Table 5.5 $[n(C_{ij}) \geq n(r) + n(c) - 1]$ (6=6)

Step-II optimality test for better improvement of indices (by drawing rectangular loops)

For example: find index value for unallocated cell O_1D_2 i.e., $O_1D_2 = 3.5 - 8.5 - 5.5 - 2.5 = -2$.

Similarly the rest of value can be calculated.

Table 5.9

SL	Unallocated cell	Closed Loop path	index
1	O_1D_2	$O_1D_2 \rightarrow O_3D_2 \rightarrow O_3D_1 \rightarrow O_1D_1$	**-2**
2	O_1D_3	$O_1D_3 \rightarrow O_3D_3 \rightarrow O_3D_1 \rightarrow O_1D_1$	-1
3	O_1D_4	$O_1D_4 \rightarrow O_3D_4 \rightarrow O_3D_1 \rightarrow O_1D_1$	1
4	O_2D_1	$O_2D_1 \rightarrow O_2D_4 \rightarrow O_3D_4 \rightarrow O_3D_1$	4
5	O_2D_2	$O_2D_2 \rightarrow O_2D_4 \rightarrow O_3D_4 \rightarrow O_3D_2$	0
6	O_2D_3	$O_2D_3 \rightarrow O_2D_4 \rightarrow O_3D_4 \rightarrow O_3D_3$	-1

In above it shows O_1D_2 shows most negative index value, so we have to transfer 5.5 units to O_1D_2
Now the revised table 5.5 can be shown below solution be z = 123.5

Table 5.10

	D1	D2	D3	D4	Av
O1	2.5(1)	3.5(5.5)	11.5	7.5	6.5
O2	1.5	0.5	6.5	1.5(1.5)	1.5
O3	5.5(6.5)	8.5	15.5(3.5)	9.5(1)	11
Req	7.5	5.5	3.5	2.5	

The above process will continue until the unallocated cells indices values become positive
So the last iteration (fourth iteration) becomes

Table 5.11

SL	Unallocated cell	Closed Loop path	Index
1	O_1D_1	$O_1D_1 \rightarrow O_1D_3 \rightarrow O_3D_3 \rightarrow O_3D_1$	**1**
2	O_1D_4	$O_1D_4 \rightarrow O_3D_4 \rightarrow O_3D_3 \rightarrow O_1D_3$	**2**
3	O_2D_1	$O_2D_1 \rightarrow O_2D_3 \rightarrow O_3D_3 \rightarrow O_3D_1$	**5**
4	O_2D_2	$O_2D_2 \rightarrow O_1D_2 \rightarrow O_1D_3 \rightarrow O_2D_2$	**0**
5	O_2D_4	$O_2D_4 \rightarrow O_3D_4 \rightarrow O_3D_3 \rightarrow O_2D_3$	**0**
6	O_3D_2	$O_3D_2 \rightarrow O_3D_3 \rightarrow O_2D_3 \rightarrow O_2D_2$	**1**

So all indices are becomes positive so optimality table be

Table 5.12

	D1	D2	D3	D4
O1	2.5	3.5(5.5)	11.5(1)	7.5
O2	1.5	0.5	6.5(1.5)	1.5
O3	5.5(7.5)	8.5	15.5(1)	9.5(2.5)

So the optimal solution be z = (3.5*5.5) + (11.5*1) + (6.5*1.5) + (5.5*7.5) + (15.5*1) + (9.5*2.5) = **121**

Case-ii: for going to method – II the solution for IBFS is obtained as **z = 121** units. (Table 5.7)
It shows that the IBFS & Optimal solution are the same by applying the Fuzzy stepping stone method (since in table 5.12 is similar to table 5.7).

6 Conclusion

The proposed Fuzzy New Method gives IBFS the optimal value for a fuzzy transportation problem that gives better results than Fuzzy Vogel Approximation method.

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