Demonstration Lecture on the expansion of $(a+b+c+d+e+\ldots\ldots)^n$ using Symmetry and principles of homogeneity of the sum of "m" terms raised to the power of a +ve integer "n"

Prof. C. Santhamma

Emeritus Professor, Centurion University, Visakhapatnam

Abstract: A novel method for the expansion of (a+b+c+d+e+…….. m terms)$^{n\ (+ve\ integer)}$ raised to the power of any +ve integer is carried out using the basic principles laid down by Jagadguru Sankaracharya of Puri Mutt (Sri Bharati Krishna Tirtha Maharaja). The procedure adopted and the results obtained will be demonstrated considering seven elements sum being raised to positive integer four. This finds application in determining roots of numbers including decimals and also of the imperfect roots. The method is applicable in the determination of roots of polynomials and is most general for any number of elements the sum of which being raised to any +ve integer.

Introduction

Jagadguru Shankaracharya Sri Bharati Krishna Tirtha Swamiji of Puri Mutt, in his series of works on Vedic Mathematics consisting of application of 29 Sutras solved many problems in Algebra, Geometry, Conics, determination of higher order roots for numbers, extending them to polynomials, recurring decimals, Squarings, Cubing, new theories such as Duplex concept, Straight Division method, theory of Osculators, Auxiliary fractions, concept of Vinculum methods, Solving higher order equations, factorization by differentiation, different types of equations, integration, differentiation, partial fractions, partial derivatives, etc. The author has extensively worked on these methods and published the results in detail into 5 Volumes as the Lecture Notes on his methods and compared them with the present day method in vogue[2]. This paper deals with the demonstration of the general expansion of the addition of seven elements raised to the positive integer four. Such expansions have application in the problems related to the root, determination of number (perfect, imperfect and decimals, polynomials, and also in determining contribution of terms of expansion pertaining to a particular decimal position.

Demonstration

A novel method is developed to arrive at the expansion of
(a+b+c+d+e+…….. m terms)$^{n\ (+ve\ integer)}$ inclusive of the coefficients of each term. This is demonstrated to the sum of seven terms raised to the power of n=4 leading to the most general expansion of m terms and n being any +ve integer, together with the corresponding coefficient of each term in the final expansion.

This method is applicable

(1) Straight Division method
(2) In the determination of decimal of any order.
(3) Solving roots of numbers and polynomials
(4) Factorisation of polynomials;

as explained by Jagadguru Sankaracharya, Shri Bharathi Krishna Tirtha Swamiji of Puri Mutt.

Expansion of any number of terms raised to any power of positive integer can be carried out using the basic principles laid down by Shri Bharathi Krishna Tirtha Swamiji. The application of such expansion in one case is now demonstrated.

Considering the Expression as $(a+b+c+d+e+f+g)^{4\ =\ Power}$

$(a+b+c+d+\ldots\ldots\ldots)^n$ ---- General

No of terms in the expansion = $\frac{7X8X9X10}{4X3X2X1} = 210$

Table 1

$(a+b+c+d+e+f+g)^4$ =

I Set	1 (Q_1)	a^4, b^4, c^4, d^4, e^4, f^4, g^4							7 terms
II Set	4(Q_2)	a^3b, ab^3, a^3c, ac^3, a^3d, ad^3, a^3e, ae^3, a^3f, af^3, a^3g, ag^3, b^3c, bc^3, b^3d, bd^3, b^3e, be^3, b^3f, bf^3, b^3g, bg^3, c^3d, cd^3, c^3e, ce^3, c^3f, cf^3, c^3g, cg^3, d^3e, de^3, d^3f, df^3, d^3g, dg^3, e^3f, ef^3, e^3g, eg^3, f^3g, fg^3							42 terms
III Set	6(Q_3)	a^2b^2, a^2c^2, a^2d^2, a^2e^2, a^2f^2, a^2g^2, b^2c^2, b^2d^2, b^2e^2, b^2f^2, b^2g^2, c^2d^2, c^2e^2, c^2f^2, c^2g^2, d^2e^2, d^2f^2, d^2g^2, e^2f^2, e^2g^2, f^2g^2							21 terms
IV Set	12(Q_4)	a^2bc	b^2cd	c^2de	d^2ef	e^2fg	f^2ag	g^2af	105 terms
		a^2bd	b^2ce	c^2df	d^2eg	e^2af	f^2bg	g^2ab	
		a^2be	b^2cf	c^2dg	d^2fg	e^2ag	f^2cg	g^2ac	
		a^2bf	b^2eg	c^2ef	d^2ab	e^2ab	f^2dg	g^2ad	
		a^2bg	b^2de	c^2eg	d^2ac	e^2ac	f^2eg	g^2ae	
		a^2cd	b^2df	c^2fg	d^2ae	e^2ad	f^2ab	g^2bc	
		a^2ce	b^2dg	c^2ad	d^2af	e^2bc	f^2ac	g^2bd	
		a^2cf	b^2ef	c^2ae	d^2ag	e^2bd	f^2ad	g^2be	
		a^2cg	b^2eg	c^2af	d^2be	e^2bf	f^2ae	g^2bf	
		a^2de	b^2fg	c^2ag	d^2bf	e^2bg	f^2bc	g^2cd	
		a^2df	b^2ac	c^2bd	d^2bg	e^2cd	f^2bd	g^2ce	
		a^2dg	b^2ad	c^2be	d^2ce	e^2cf	f^2be	g^2cf	
		a^2ef	b^2ae	c^2bf	d^2cf	e^2cg	f^2cd	g^2de	
		a^2eg	b^2af	c^2bg	d^2cg	e^2df	f^2ce	g^2df	
		a^2fg	b^2ag	c^2ab	d^2bc	e^2dg	f^2de	g^2ef	
		15 terms	15 terms	15 terms	15 terms	15 terms	15 terms	15 terms	
V Set	24 (Q_5)	abcd, abce, abcf, abcg, abde, abdf, abdg, abef, abeg, abfg, acde, acdf, acdg, acef, acfg, aceg, adef, adeg, adfg, aefg, bcde, bcdf, bcdg, bcfg, bcef, bceg, bdef, bdeg, bdfg, befg, cdef, cdeg, cdfg, cefg, defg							35 terms
		TOTAL							210 terms

From the above table, one can get information on the contribution of the terms which are responsible for a particular decimal point.

The procedure is as follows :

In the expression $(a+b+c+d+e+f+g)^4$ one has to consider "a" as the first non decimal point. "b", "c", "d", "e", "f" & "g" are successively the first, second, third, fourth, fifth and sixth decimal points. To evaluate a particular expression, say for example, c^2e^2, the position of "c" is 2 and the position of "e" is 4. Therefore c^2e^2 is 12.

Similarly, one can evaluate the expressions that contribute to any decimal position.

Coefficients by Combination Method

Table 2

a^4	nc_4	1	Q_1
a^3b	nc_1 x $(n-1)c_3$	4	Q_2
a^2b^2	nc_2 x $(n-2)c_2 \equiv nc_3$ x $(n-3)c_1$	6	Q_3
a^2bc	nc_1 x $(n-1)c_1$ x $(n-1-1)c_2 \equiv nc_2$ x $(n-1)c_1x(n-3)c_1$	12	Q_4
abcd	nc_1 $x(n-1)c_1$ x $(n-2)c_2$ x $(n-3)c_3 \equiv (n-1)c_1x(n-1-1)c_2x(n-1-1-1)c_3$	24	Q_5

Coefficients of each set Q_1, Q_2, Q_3, Q_4, Q_5

The expansion is
Q_1 (I Set Sum) + (4) Q_2 (II Set Sum) + (6) Q_3 (III Set Sum) + (12) Q_4 (IV Set Sum) + (24) Q_5 (V Set Sum)

A method is derived for the first time to give out the expansion terms as explained by Shri Bharati Krishna Tirtha Swamiji. This can be extendable for any number of terms to any power.

Consider the number as abcdefg and its expansion to the power 4

$$(a+b+c+d+e+f+g)^4$$

The expansion consists of different groups (sets) of terms following the principle of Symmetry, and homogeneity. The different sets consist of single element, combination of two elements, combination of three elements, combination of four elements satisfying the symmetry and homogeneity. These are shown in Table 1 under 5sets. The coefficients of the terms in each set could be evaluated using combination theory. The results of such are given in the Tabel 2. The final expansion is
Q_1Set1(sum of terms)+Q_2Set2(sum of terms)+Q_3Set3(sum of terms)+Q_4Set4(sum of terms)+Q_5Set5(sum of terms)+ ….

Application in identifying the contribution of terms to a particular decimal accuracy.

The positions of a b c d e f g in the given number are $10^0,10^1,10^2,10^3,10^4,10^5,10^6$ respectively. Keeping this in view, the decimal contribution of terms in the expansion upto 6th decimal is as follows (Table 3).

Table 3

1st Decimal	a^3b. b is calculated from the Common Divisor using Swamiji's Straight Division method.
2nd decimal	a^3c, a^2b^2
3rd decimal	a^3d,a^2bc,ab^3
4th decimal	a^3e, a^2c^2, a^2bd, b^2ac, b^4, b^3c
5th decimal	a^3f, a^2cd, b^2ad, a^2be, c^2ab, b^3c, a^2be
6th decimal	a^3g, a^2d^2, a^2bf,a^2ce, b^2ae, abcd, c^3, b^2c^2, b^3d
12th decimal	d^4,ae^3, c^3g, $a^2g^2,b^2f^2,c^2e^2,b^2eg,c^2ef,d^2ag,d^2bf,d^2ce,fab$

Similary one can read from the terms the contributions to various other decimals

Verification $(c+b+a)^3$

Consider the number (3 digits)

c	b	a
3	2	1

$(c+b+a)^3$ a – 10^0, b – 10^1, c – 10^a

On the basis of the present procedure, the terms are

a^3	$3a^2b$	$3ab^2+3a^2c$	b^3+6abc	$3ac^2+3b^2c$	$3bc^2$	c^3
10^0	10^1	10^2	10^3	10^4	10^5	10^6

a^3	=	1
$3a^2b$	=	60
$3ab^2$	=	1200
$3a^2c$	=	900
b^3	=	8000
6abc	=	36000
$3ac^2$	=	270000
$3b^2c$	=	360000
$3bc^2$	=	5400000
c^3	=	27000000
$(321)^3$	=	33076161

$(321)^2$	=	10341
321	X	321
$(321)^3$	=	33076161

References

1. Vedic Mathematics by Jagadguru Swamiji, Sri Bharati Krishna Tirthaji Maharaja, Publishers : Motilal Banarsidas Pvt. Ltd, New Delhi
2. Lecture Notes (in 5 Volumes) by Prof. C. Santhamma et.al published by Bharateeya Vidya Kendram, Visakhapatnam