Design of Repetitive Acceptance Sampling Plan for Truncated Life Test using Inverse Weibull Distribution

Navjeet Singh[1], Navyodh Singh[2], Harpreet Kaur[3]

[1]Department of Mathematics, Sant Baba Bhag Singh University, Jalandhar, India
[2]Department of Mathematics, Sant Baba Bhag Singh University, Jalandhar, India
[3]Department of Mathematics, Sant Baba Bhag Singh University, Jalandhar, India

E-mail: [1]navjeet8386@yahoo.com, [2]navyodh81@yahoo.co.in

Abstract

In this paper, we present a repetitive acceptance sampling plan for an inverse Weibull distribution based on truncated life test with known shape parameter. The design parameters such as sample size and acceptance number are formulated by considering the median life time of the test units as a quality parameter. The values of the design parameter are obtained under the constraint of two risks, known as the producer's risk and consumer's risk at a certain level. We present a detailed study to assist the proposed methods with the help of tables at different values of the known parameter. A comparison is done between the proposed plan and the single acceptance plan. The implementation of the proposed plan is explained with the help of real-life data.

Keyword: Consumer's risk, repetitive acceptance sampling plan, single acceptance sampling plan, producer's risk, truncated life test, quality level.

1. Introduction

The quality of the product is the most important aspect of the production field. So the manufacturer's center of attention is the quality of the product which helps to build the reputation of the firm. Scrutiny of the product is also mandatory to safeguard the product quality before the product enters into the market. However, investigation of all the items of the product may not be possible due to time restriction, survey error, and cost. In such cases, the maker uses acceptance sampling inspection which is one of the major element of statistical quality control. In acceptance sampling, only a few items are chosen at random from the submitted lot and according to the results of random sample items, the entire lot is either accepted or rejected. Acceptance sampling plans have become very useful in today's fast-growing industrial environment to check the life time of a product. Quality of any product is a very important feature, both for the producer and the consumer. Whatever acceptance sampling plans have implemented, the producer's and consumer's risks are always associated with the lot decision. The probability of rejecting a good lot is called producer's risk (α) and the chance of accepting a bad lot is called consumer's risk (β). A sampling plan works well if both the risks are reduced and it saves the time and money both. Broadly the acceptance plans are of two types. These are attribute sampling plans and variable sampling plans. When the decision is taken on basis of failure times then it is called variable sampling plan and when the decision is taken on the number of failures in the observed sample then it is called an attribute sampling plan. Attribute sampling plans have some advantages over the variable sampling plans. The various assumptions on which variable sampling plans are based are not only difficult to meet but sometimes even may not be known. Also, it is more expensive than the attribute sampling plans. We will be dealing with attribute sampling plan where a selected lot will be put under the experiment known as life testing. Life testing is a process which can be used to evaluate the lastingness of a product under some deliberate conditions. In addition, the life test can be conducted for the pre-assigned time and such type of life test is called as truncated life test. One of the most uncomplicated truncated life tests is the single acceptance sampling plan which has two design parameters n (sample size) and c (acceptance number). In single acceptance sampling plans, a random sample of n units is placed on a life test for pre-decided time t_0. Now our main task is to accept or reject the whole lot from which the sample is taken. If during the test total number of observed failures is less than or equal to c then the lot is accepted otherwise the experiment is stopped and the lot is rejected. Many researchers have done studies on the truncated life test for various statistical distributions. Single acceptance sampling plans have been developed by [1] for Weibull distribution and by [2] for normal and log-normal distribution. Carrying on with the identical approach by [3] studied the double sampling for assuring the mean life of the product based on truncated life tests under Maxwell distribution. [4] worked on double acceptance sampling plans with zero one failure proposal, in which a lot is accepted if no failures are observed from the first sample and it is rejected if two or more failure is observed. These researchers developed the plan for generalized log-logistic distribution.

According to [5] classification of acceptance sampling plans, special purpose sampling plans dominate a lot. Repetitive group sampling plan (RGS), multiple deferred state (MDS) sampling plan number among the special purpose plans. Sherman [6] initiated another sampling plan called repetitive acceptance sampling plan (RASP) which is an attribute sampling plan. According to him, this plan gave results for the sample size which are less than an optimal sample size given by the single acceptance sampling plan. Many researchers have compared these two sampling plans. Recently [7] designed RASP for generalized inverted exponential distribution based on truncated life test and found that it gives better results than single acceptance sampling plan. In the early days of research on acceptance sampling plans mean life time was used to ensure the standard or authenticity of products. However, the acceptance sampling plans based on mean life time were not considered for the engineering designs. In this paper, we are using median life time as a quality level. In this paper, we have developed the repetitive acceptance sampling plan based on inverse Weibull distribution. The rest of the paper is organized as follows. Introduction of the inverse Weibull distribution is given in section 2. The design of the repetitive sampling plan in section 3. Also, the performance measures and affectability investigation under Inverse Weibull distribution are discussed and related illustrative example along with a numerical comparison between single acceptance plan and repetitive acceptance plan is given in this section. Practical application is discussed along with the selection criterion for the shape parameter in section 4. Finally, a conclusion is given in section 5.

2. Inverse Weibull Distribution

Weibull distribution is one of the most important life time distribution and also it is a general case of exponential and Rayleigh distribution. Weibull distribution gained popularity due to the different shapes it can assume by varying its parameters. Weibull distribution has two parameters, but in many practical problems, it is not unjustified to assume one of them, the shape parameter to be known and perform the necessary examination under the assumption that only the scale parameter is unknown. Although Weibull distribution has found wide applications in the field of analysis of material strength, software reliability and the reliability of evaluation of power systems. But in some cases, inverse Weibull distribution may be a more appropriate model to analyze the life time data. The inverse Weibull (IW) distribution is life time distribution which has found wide spread applications in various areas of reliability analysis discipline. The IW model has been derived as a suitable model for describing the degradation phenomena of mechanical components such as dynamic components of diesel engines (i.e. pistons, crankshafts, main bearings etc.) with respect to other distributions considered (exponential and two-parameter Weibull) see for example [8]. The inverse Weibull distribution has received a lot of attention in the literature. [9] studied the shapes of the density and failure rate functions for the basic inverse model and have derived four alternative failure models for mechanical components and systems subject to degradation phenomena such as wear, fatigue, and corrosion. In particular, the two-parameter inverse Weibull model seemed to be, on the basis of theoretical considerations, a suitable model to describe wear and mechanical degradation phenomena. If the random variable Y has the Weibull distribution then the probability density function is given by

$$f_Y(y;\delta,\lambda) = \delta\lambda y^{\delta-1}e^{-\lambda y^{\delta}} \ , y > 0 \tag{1}$$

then the random variable $T = \frac{1}{Y}$ has the IW distribution then the probability density function is given by

$$f_T(t;\ \delta,\lambda) = \delta\lambda e^{-\lambda t^{-\delta}} t^{-(\delta+1)} \ , t > 0 \tag{2}$$

The quantities $\delta > 0$ and $\lambda > 0$ are the shape and scale parameters respectively. If T follows IW model cumulative distribution function of T is given by

$$F_T(t;\ \delta,\lambda) = e^{-\lambda t^{-\delta}}, t > 0 \tag{3}$$

The K-th ($K \leq \delta$) moment of T is

$$E\left(T^k\right) = \lambda^{\frac{k}{\delta}}\,\Gamma\left(1 - \frac{k}{\delta}\right) \tag{4}$$

and for $k > \delta$ the moment do not exist. It is clear that it is a heavy tail distribution and as $\delta \to \infty$, the tail probability decreases. For $0 < \delta \leq 1$, the mean does not exist and for $1 < \delta \leq 2$, the mean exists but variance does not exist.

Median of the inverse Weibull distribution is given by

$$m = \left(\frac{\lambda}{log2}\right)^{\frac{1}{\delta}} \tag{5}$$

3. Design of Repetitive sampling plan

Suppose the median life time of units is denoted by m_0. Now we want to check whether the true median life time m of a unit is greater than specified life time m_0. The submitted lot is considered to be good if $m \geq m_0$ and if it does not holds then the lot is rejected. Before we start the experiment it is convenient to write the experiment time $t_0$in terms of experiment termination ratio a and specified values of the median life m_0, given by $t_0 = am_0$.

3.1 Operating procedure

The operating procedure of repetitive acceptance sampling plan (RASP) under truncated life test is described as follows:

Step-I. Sample of size n is selected at random from a submitted lot and put on a life test, separately, until a specified time t_0.

Step-II. Lot is accepted if the number of failures by the time t_0, D, is smaller than or equal to c_1 (which is called the acceptance number). Truncate the test and reject the lot as soon as the number of failures exceeds c_2, where $c_2 \geq c_1$.

Step-III. If$c_1 < D \leq c_2$, then go to step I and experiment is repeated.

The above attributes repetitive sampling plan has three parameters namely n, c_1 and c_2. When $c_1 = c_2$ then this plan reduces to ordinary single acceptance sampling plan. The purposed RASP repeatedly utilize an individual sample to reach a conclusive decision.

3.2 Performance measures

The performance of any examining arrangement can be researched by its performance measures. The probability of lot acceptance is determined by using the Operating Characteristic (OC) function, which is derived to be:

$$P_A(p) = \frac{P_a(p)}{P_a(p) + P_r(p)} \ ; 0 < p < 1. \tag{6}$$

where p is the probability that a product under test fails before t_0, P_a is the probability of acceptance of a submitted lot and P_r is the corresponding probability of lot rejection. These probabilities can be calculated as follows

$$P_a(p) = P(D \leq c_1|p) = \sum_{i=0}^{c_1}\left(\binom{n}{i} p^i\,(1-p)^{n-i}\right) \tag{7}$$

and

$$P_r(p) = P(D > c_2|p) = 1 - \sum_{i=0}^{c_2}\left(\binom{n}{i} p^i\,(1-p)^{n-i}\right) \tag{8}$$

where D denotes the number of failures by time t_0. Then the OC function in the equation is represented by

$$P_A(p) = \frac{\sum_{i=0}^{c_1}\left(\binom{n}{i} p^i\,(1-p)^{n-i}\right)}{1-\sum_{i=0}^{c_2}\left(\binom{n}{i} p^i\,(1-p)^{n-i}\right)+\sum_{i=0}^{c_1}\left(\binom{n}{i} p^i\,(1-p)^{n-i}\right)} \tag{9}$$

where $p = (t_0; \delta, \lambda)$ is the probability that a test unit fails before the termination time point t_0 such that $p = e^{-\frac{\lambda}{t_0{}^{\delta}}}$ However using $t_0 = am_0$ and equation (5), p can be written as

$$p = e^{\left(-\frac{m}{m_0}\right)^{\delta} log2a^{-\delta}} \tag{10}$$

Now to calculate p, the value of a, m, m_0 and the shape parameter δ should be known in advance. However, the median lifetime of product depends on two values the shape parameter and scale parameter see equation (5). Quality level is measured as a ratio of $\frac{m}{m_0}$. Now, when $m > m_0$, the producer needs that the probability of rejecting a lot should be smaller than the probability of rejecting a good lot. On the other hand from a consumer's perspective probability of accepting a lot should be smaller than the probability of accepting a bad lot when, $m = m_0$. Now we will calculate the design parameters (n, c_1, c_2) of the proposed plan at a given quality level $\frac{m}{m_0}$, corresponding to consumer's risk β and producer's risk α, by solving the following two inequalities simultaneously:

$$P_a\left(p_1|\frac{m}{m_o} = r_1\right) \leq \beta \tag{11}$$

$$P_a\left(p_2|\frac{m}{m_o} = r_2\right) \geq 1-\alpha \tag{12}$$

where p_1 is the probability of failure before the termination time t_0 corresponding to the quality level $\frac{m}{m_0} = 1$ and p_2 is the probability of failure corresponding to the quality level $\frac{m}{m_0}$ = 2, 3, 4, 5, 6. Now more than one set of values for the design parameters will be obtained using equation (10), (11) and (12). So we will take that help of the average sample number (ASN) to obtain the minimum sample size. For the proposed RASP the minimum ASN is required to make a decision to accept or reject the lot and it is given by:

$$ASN(p) = \frac{n}{P_a(p)+P_r(p)} \tag{13}$$

As a result, the desired design parameters can be obtained by solving the optimization problem as defined below:

$$\text{Minimize ASN} = \frac{n}{P_a+P_r}$$

$$P_a\left(p_1 | \frac{m}{m_o} = r_1\right) \leq \beta$$

$$P_a\left(p_2 | \frac{m}{m_o} = r_2\right) \geq 1-\alpha$$

where n is an integer.

3.3 Affectability investigation under Inverse Weibull distribution

The design parameters satisfying the equation (11), (12), (13) are reported in table 1 for δ = 0.75. We are calculating the parameters at three different levels of test termination ratio a = 0.5, 0.7, 1.0, four different levels of consumer's risk β = 0.25, 0.10, 0.05, 0.01 and only one value of producer's risk α as 0.05. We consider the quality level $r_1 = 1$ at consumer's risk , while five values of quality level r_2 at the producer's risk as 2, 3, 4, 5 and 6. The average sample number (ASN) is also reported in table 1 and table 2. The lot acceptance probability risk(p_α) at producer's risk and consumer's risk (p_β) are also given in table 4. Calculated values indicate that with increase in the experiment termination ratio the sample size and ASN increases and it is only visible when the median ratio r_2=2, though it is not uniform for this value of median ratio also. For example when β = 0.25 and a = 0.5 then at quality level r_2=2 sample size is 18 and ASN is 27.70 and it increases to sample size 24 and ASN 32.68 at termination ratio a = 1.0. An increase in sample size and ASN is observed when the consumer's risk is decreased. Similar increase is witnessed in table 2 with δ =1.25. But when we compare the values of sample size and ASN for different shape parameter, we find that smaller sample size and ASN are observed with large shape parameter. In table 2 we notice that with increase in value of time termination multiplier a the sample size and ASN decrease sharply. A comparison between RASP and single acceptance sampling plan is given in table 3. In field work, smaller sample sizes are preferred. From the table 3 it clearly reflects that RASP gives smaller sample size than single acceptance sampling plans. However, this is not true in cases where no failure is allowed when acceptance number $c = 0$ in single acceptance sampling. Such a plan is called zero acceptance sampling plan. We get the idea of probability of acceptance at producer's risk(p_α) and consumer's risk (p_β) when there is increment in median ratio and for different values of shape parameter δ and a in table 4.

Table 1: Minimum of average sample number with $\delta = 0.75$

β	r_2	a=0.5				a=0.7				a=1.0			
		c_1	c_2	n	ASN	c_1	c_2	n	ASN	c_1	c_2	n	ASN
0.25	2	3	5	18	27.7	5	7	19	27.88	9	11	24	32.68
	3	2	2	12	12.0	0	2	5	12.50	3	4	10	12.57
	4	0	1	5	7.68	0	1	4	6.07	1	2	6	7.83
	5	0	1	5	7.68	1	1	6	5.99	0	1	3	4.80
	6	0	0	4	4.00	1	1	6	5.99	0	1	3	4.80
0.10	2	3	6	22	36.1	4	8	21	39.91	7	11	23	42.08
	3	0	2	9	14.9	1	3	10	14.87	3	5	13	17.20
	4	0	1	7	9.11	1	2	9	10.67	0	2	5	9.41
	5	0	1	7	9.11	0	1	6	7.33	1	2	7	8.37

	6	0	1	7	9.11	0	1	6	7.33	0	1	4	5.33
0.05	2	2	6	21	40.4	6	10	28	43.24	7	12	25	47.92
	3	1	3	15	19.5	2	4	14	18.21	2	5	12	18.98
	4	0	2	10	14.8	0	2	8	11.27	1	3	9	11.75
	5	0	1	9	10.4	0	2	8	11.27	0	2	6	8.93
	6	0	1	9	10.4	0	1	7	8.01	0	2	6	8.93
0.01	2	4	9	35	50.1	9	14	42	53.28	13	19	43	58.45
	3	0	3	15	20.2	2	5	18	22.24	3	7	18	23.57
	4	0	2	13	15.6	0	3	12	15.27	1	4	12	14.82
	5	0	1	13	13.6	0	2	10	11.83	0	3	9	12.03
	6	0	1	13	13.6	0	1	10	10.39	0	2	8	9.30

Table 2: Minimum of average sample number with $\delta = 1.25$

β	r_2	a=0.5				a=0.7				a=1.0			
		c_1	c_2	n	ASN	c_1	c_2	n	ASN	c_1	c_2	n	ASN
0.25	2	0	1	9	13.10	2	2	11	11.0	1	3	7	12.44
	3	0	0	7	7.00	0	1	5	7.39	0	1	3	4.79
	4	0	0	7	7.00	0	0	4	4.00	0	1	3	4.79
	5	0	0	7	7.00	0	0	4	4.00	0	0	2	2.00
	6	0	0	7	7.00	0	0	4	4.00	0	0	2	2.00
0.10	2	0	1	12	15.38	0	2	8	13.7	2	4	10	14.75
	3	0	0	11	11.00	0	1	7	8.75	0	1	4	5.33
	4	0	0	11	11.00	0	0	6	6.00	0	1	4	5.33
	5	0	0	11	11.00	0	0	6	6.00	0	1	4	5.33
	6	0	0	11	11.00	0	0	6	6.00	0	0	4	4.00
0.05	2	0	1	15	17.54	1	3	13	17.8	3	5	14	17.14
	3	0	0	15	15.00	0	1	8	9.41	0	2	6	8.92
	4	0	0	15	15.00	0	0	8	8.00	0	1	5	5.92
	5	0	0	15	15.00	0	0	8	8.00	0	1	5	5.92
	6	0	0	15	15.00	0	0	8	8.00	0	0	5	5.00
0.01	2	0	2	23	26.92	0	3	13	18.6	2	6	15	21.42
	3	0	0	22	22.00	0	1	12	12.5	0	2	7	8.95
	4	0	0	22	22.00	0	0	12	12.0	0	1	7	7.40
	5	0	0	22	22.00	0	0	12	12.0	0	1	7	7.40
	6	0	0	22	22.00	0	0	12	12.0	0	1	7	7.40

Table 3: Comparison between RASP and single acceptance sampling plan

β	r_2	δ=0.75						δ =1.25				
		a=0.5		a=0.7		a=1.0		a=0.5		a=0.7		a=1.0
		ASN	n(c)	ASN	n(c)	ASN	n(c)	ASN	n(c)	ASN	n(c)	ASN
0.25	2	27.70	34(8)	27.88	36(12)	32.68	40(17)	13.10	14(1)	11.0	11(2)	12.44
	3	12.0	12(2)	12.50	15(4)	12.57	14(5)	7.0	7(0)	7.39	8(1)	4.49
	4	7.68	8(1)	6.07	9(2)	7.83	10(3)	7.0	7(0)	4.0	4(0)	4.79
	5	7.68	8(1)	5.99	6(1)	4.8	7(2)	7.0	7(0)	4.0	4(0)	2.0
	6	4.0	4(0)	5.99	6(1)	4.8	5(1)	7.0	7(0)	4.0	4(0)	2.0
0.10	2	36.13	51(11)	39.91	52(16)	42.08	59(24)	15.38	26(2)	13.75	18(3)	14.75
	3	14.91	20(3)	14.87	21(5)	17.20	24(8)	11.0	11(0)	8.75	10(1)	5.33
	4	9.11	16(2)	10.67	15(3)	9.41	14(4)	11.0	11(0)	6.0	6(0)	5.33
	5	9.11	11(1)	7.33	12(2)	8.37	12(3)	11.0	11(0)	6.0	6(0)	5.33
	6	9.11	11(1)	7.33	9(1)	5.33	9(2)	11.0	11(0)	6.0	6(0)	4.0
0.05	2	40.49	66(14)	43.24	67(20)	47.92	76(30)	17.54	31(2)	17.86	25(4)	17.14
	3	19.59	27(4)	18.21	27(6)	18.98	28(9)	15.0	15(0)	9.41	12(1)	8.92
	4	14.87	18(2)	11.27	17(3)	11.75	18(5)	15.0	15(0)	8.0	8(0)	5.92
	5	10.47	14(1)	11.27	14(2)	8.93	13(3)	15.0	15(0)	8.0	8(0)	5.92
	6	10.47	14(1)	8.01	10(1)	8.93	11(2)	15.0	15(0)	8.0	8(0)	5.0

0.01	2	50.17	96(19)	53.28	98(28)	58.45	107(41)	26.92	41(2)	18.65	35(5)	21.42
	3	20.25	38(5)	22.24	38(8)	23.57	42(13)	22.0	22(0)	12.53	17(1)	8.95
	4	15.65	29(3)	15.27	25(4)	14.82	27(7)	22.0	22(0)	12.0	12(0)	7.40
	5	13.62	24(2)	11.83	21(3)	12.03	19(4)	22.0	22(0)	12.0	12(0)	7.40
	6	13.62	19(1)	10.39	18(2)	9.30	17(3)	22.0	22(0)	12.0	12(0)	7.40

Table 4: Probabilities of acceptance at consumer's risk and producer's risk

β	r_2	δ=0.75						δ=1.25					
		a=0.5		a=0.7		a=1.0		a=0.5		a=0.7		a=1.0	
		p_α	p_β	p_α	p_β	p_α	p_β	p_α	p_β	p_α	p_β	p_α	p_β
0.25	2	0.9595	0.2143	0.9537	0.2259	0.9511	0.2093	0.9848	0.21531	0.9542	0.2227	0.9535	0.1110
	3	0.9530	0.2251	0.9671	0.1877	0.9592	0.2162	0.9895	0.2242	0.9979	0.1870	0.9802	0.1999
	4	0.9848	0.2369	0.9575	0.1914	0.9521	0.1428	0.9993	0.2242	0.9912	0.1912	0.9957	0.1999
	5	0.9956	0.2369	0.9691	0.2268	0.9642	0.2000	0.9999	0.2242	0.9988	0.1912	0.9649	0.2499
	6	0.9551	0.2240	0.9866	0.2268	0.9828	0.2000	0.9999	0.2242	0.9998	0.1912	0.9824	0.2499
0.10	2	0.9581	0.0887	0.9526	0.0645	0.9516	0.0852	0.9886	0.0988	0.9664	0.0628	0.9616	0.0806
	3	0.9611	0.0572	0.9547	0.0652	0.9549	0.0610	0.9544	0.0954	0.9957	0.0689	0.9601	0.0833
	4	0.9679	0.0949	0.9690	0.0797	0.9544	0.0588	.0.9544	0.0954	0.9868	0.0836	0.9914	0.0833
	5	0.9908	0.0949	0.9601	0.0546	0.9719	0.0747	0.9544	0.0954	0.9982	0.0836	0.9980	0.0833
	6	0.9971	0.0949	0.9841	0.0546	0.9653	0.0833	0.9544	0.0954	0.9997	0.0836	0.9652	0.0624
0.05	2	0.9517	0.0402	0.9549	0.0439	0.9502	0.0414	0.9532	0.0475	0.9818	0.0486	0.9516	0.0351
	3	0.9759	0.0373	0.9668	0.0483	0.0960	0.0305	0.9777	0.0406	0.9943	0.0430	0.9888	0.0232
	4	0.9927	0.0353	0.9648	0.0223	0.9583	0.0255	0.9986	0.0406	0.9825	0.0365	0.9857	0.0370
	5	0.9841	0.0402	0.9922	0.0223	0.9724	0.0232	0.9999	0.0406	0.9976	0.0365	0.9967	0.0370
	6	0.9951	0.0402	0.9776	0.0305	0.9910	0.0232	0.9999	0.0406	0.9996	0.0365	0.9567	0.0312
0.01	2	0.9571	0.0087	0.9530	0.0096	0.9543	0.0093	0.9840	0.0086	0.9630	0.0066	0.9669	0.0052
	3	0.9502	0.0049	0.9660	0.0092	0.9608	0.0049	0.9675	0.0091	0.9864	0.0073	0.9802	0.0099
	4	0.9824	0.0093	0.9729	0.0025	0.9629	0.0039	0.9980	0.0091	0.9739	0.0069	0.9697	0.0082
	5	0.9654	0.0081	0.9830	0.0066	0.9803	0.0026	0.9999	0.0091	0.9964	0.0069	0.9930	0.0082
	6	0.9892	0.0081	0.9518	0.0058	0.9742	0.0045	0.9999	0.0091	0.9995	0.0069	0.9983	0.0082

Example

Suppose that producer submits a lot of units and claim that specified life time of units is 2000 hours, here assume that life time of the unit follows an inverse Weibull distribution with shape parameter 0.75. Further, consider the consumer's risk 5% when true median life of units is 2000 hours and producer's risk 5% when the true median life of units is 6000 hours. Now we concerned about the design parameters of the repetitive acceptance sampling plan when an experimenter would like to run life test experiment for 1400 hours. Notice that in this case we have γ = 0.75, m = 2000, a =0.7, β = 0.05, r_1 = 1, α = 0.05 and r_2 = 3. Subsequently, the design parameters from table 1 can be obtained as (c_1, c_2) = (2, 4) and n=14 with ASN 18.21. The ramification of this observation is that consumer will take a random sample of size 14 units from the purposed lot and then will subject to a life test for 1400 hours. During the experiment, if 2 or less unit fails then the lot will be accepted and if more than 4 units fails then the lot will be rejected. If the number of failed units are more than 2 and less than equal to 4 the repeat the experiment. Therefore to make a decision whether to accept or reject the proposed lot, an average of 18.21 number of units are required under the plan. Now if we compare the RASP with the traditional single acceptance plan then the design parameters from the table 3 are n=27 and c=6. It make a point that a random sample of size n=27 units should be subjected to a life test for about 1400 hours. If during the experiment more than 6 units fail then that lot will be rejected otherwise, it will be accepted. It is noticed that the RASP is more reasonable and efficient than the single acceptance sampling plan as the ASN is smaller than the sample size of single acceptance sampling plan. It is further examined that sample size and acceptance number decrease rapidly as the true median life is allowed to increase and set to 4000,6000,8000,10000 and 12000 at 5% producer's risk as shown in table below.

Proposed plan	Design parameter	$m/m_0 = r_2$				
		2	3	4	5	6
RASP	(n, c, d)	(28,6,10)	(14,2,4)	(8,0,2)	(8,0,2)	(7,0,1)
SSP	(n, c)	(67,20)	(27,6)	(17,3)	(14,2)	(10,1)

4. Industrial implementation of the repetitive acceptance sampling plan

In this section, the implementation of the purposed plan is explained where the shape parameter of life time distribution is unknown. Estimation for the shape parameter is done before the construction of any sampling plan in such situations. This situation is handled by using the past history of the production process or even using a pre-sample for which the corresponding inferences are derivable. For clear understanding, we consider a real data set to demonstrate the selection of the shape parameter. This data set is given by [10] and represents the breakdown time of electrical insulating fluid subject to a 30KV voltage stress. The corresponding break down time (in minutes) are

7.74, 17.05, 20.46, 21.02, 22.66, 43.40, 47.30, 139.07, 144.12, 175.88, 194.90

It is confirmed that the distribution of breakdown times is relevant to inverse Weibull distribution and the maximum likelihood estimate of the data is determined as 1.05. Therefore the given data set can be examined using inverse Weibull distribution. Suppose that the specified median time to breakdown of an insulating fluid is 50 minutes subject to 30KV voltage stress. The experiment time t_0 is 25 minutes.Therefore the value of termination ratio for this test is 0.5. Producer's risk is 5 %(α) = 0.05and consumer's risk is 10 % (β) = 0.10 and the median ratio is 2.0, 3.0, and 4.0. Table 5 yields the most advantageous parameters of the RASP by taking the value of shape parameter (δ) =1.05. These values can be used in real life problems to make sufficient conclusion about the given lot. The value of shape parameter was unknown so with the help of historic data we were able to calculate the value of δ. Now we are further trying to notice the effect of mis-specification on the lot acceptance probabilities for producer's risk p_α and consumer's risk p_β.Let δ_0 be the true shape parameter of the purposed lot.

Now we can re-write the value of p as $p_0 = e^{\left(-\frac{m}{m_0}\right)^{\delta_0} log2a^{-\delta_0}}$. Now the value of shape parameter are true to a fairly high degree if the values of design parameters obtained using δ still satisfy the producer's risk and consumer's risk under p_0 .Now to explicate we consider the design parameters from table 5 and calculate the acceptance probabilities for random values of δ_0 such as 0.90, 0.95, 1.0, 1.10, 1.15. The acceptance probabilities are described in table 6. From the calculated values it is perceived that producer's risk is satisfied at higher values of the true shape parameter but consumer's risk is not satisfied and it is vice versa for lower values of δ_0. This can proved to be a vital detail for execution of various sampling plans in practice.

Table 5: Acceptance probabilities under misspecification of shape parameter $\delta = 1.05$

β	r_2	n	c_1	c_2	ASN	$p\alpha$
0.10	2	12	0	2	19.67	0.9623
	3	10	0	1	12.59	0.9948
	4	9	0	0	9	0.9812

Table 6: Acceptance probabilities under misspecification of shape parameter δ

β	r_2	δ_0=0.90		δ_0=0.95		δ_0 =1.0		δ_0 =1.05		δ_0=1.10		δ_0 = 1.15	
		p_α	p_β	p_α	p_β	p_α	p_β	p_α	p_β	p_α	p_β	p_α	p_β
	2	0.7915	0.0303	0.8748	0.0387	0.9292	0.0494	0.9623	0.0627	0.9809	0.0795	0.9907	0.1003
	3	0.9525	0.0478	0.9757	0.0576	0.9883	0.0693	0.9948	0.0830	0.9977	0.0991	0.9991	0.1179
	4	0.9052	0.0558	0.9412	0.0648	0.9654	0.0750	0.9812	0.0865	0.9910	0.0993	0.9995	0.1135

5. Conclusion

In this paper, the designing of RASP has been discussed based on the time truncated life test under inverse Weibull distribution. The median life of the product is considered as a quality characteristic of the product. The optimal parameters and ASN of the RASP under the inverse Weibull distribution have been determined such that producer's risk and consumer's risk are satisfied with the minimum ASN. We have compared the ASN with the sample size of single acceptance plan and found it more economical except possibly zero acceptance sampling plan. The probability of acceptance are reported in a table 4 both for consumer's risk and producer's risk. The case of mis-specification of shape parameter has also been discussed. From this study, it is concluded that proposed plan will be effective in reducing the inspection cost and time.

References

[1] H.P. Goode, and J.H.K. Kao, "Sampling plans based on the Weibull distribution", Proceeding of the Seventh National Symposium on Reliability and Quality Control, Philadelphia. (1961), pp. 24-40.

[2] S.S. Gupta, "Life test sampling plans for normal and lognormal distributions", Technimetrics, 4(2), (1962), 151-175.

[3] Wenhao Gui, "Double acceptance sampling plan for time truncated life tests based on Maxwell distribution", American Journal of Mathematical and Management Sciences, 33, no. 2 (2014): 98-109.

[4] M. Aslam and C.H. Jun, "A double acceptance sampling plans for generalized log-logistic distribution with known shape parameter", Journal of Applied Statistics, 37(3), (2010), 405-414.

[5] Harold F. Dodge, "Notes on the Evolution of Acceptance Sampling Plans Part I", Journal of Quality Technology 1, no. sup1 (1969), 1-12.

[6] Robert E. Sherman, "Design and evaluation of a repetitive group sampling plan." Technometrics 7, no. 1 (1965), 11-21.

[7] Navyodh Singh, Navjeet Singh, Harpreet Kaur , "A Repetitive Acceptance Sampling Plan for Generalized Inverted Exponential Distribution Based on Truncated Life Test", International Journal of Scientific Research in Mathematical and Statistical Sciences, Vol.5, Issue.3,(2018), pp.58-64.

[8] D.N.P. Murthy, M. Xie, R. Jiang, Weibull Models, Wiley, New York, (2004).

[9] A.Z. Killer and A.R.R. Kamath, "Alternative reliability models for mechanical systems", Third International Conference on Reliability and Maintainability, Tousle, France. (1982) October 18-21.

[10] Lawless, J., Statistical Models and Methods for Lifetime Data. New York, Wiley, (2003).