Crude Oil Price Model

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Abstract

A Dynamical system consists of a set of possible states, together with a rule that determines the present state in terms of past states. Whereas Chaos is a state of disorder. Chaos theory deals with the long-term qualitative behavior of dynamical systems. Chaos systems whose state evolves with time that may exhibit dynamics that are highly sensitive to initial conditions which is popularly known as the butterfly effect.

Crude oil price movement has a chaotic behavior because of large fluctuations in crude oil prices. The purpose of this paper is to focus on the mathematical model of crude oil prices and analyze the complex relations among energy price, energy demand -supply and net imports. We study the chaotic dynamics behavior price differential of crude oil and secondly we analyze chaotic behavior of price differential of crude oil.

Key words : Chaos, Crude oil price, dynamical system

Introduction

Energy plays an important role in economic development. Crude Oil is an important energy commodity to mankind. It is a complex mixture consisting of more than 200 organic compounds, especially hydrocarbons mostly alkenes and smaller fraction aromatics. Two of the most important characteristics are density and sulfur content. High quality of crude oils are characterized by low density and low sulfur content. and are more expensive. Simply demand and supply theories are useful in describing oil price movements. Any fluctuation in crude oil prices affects our personal and corporate financial lives .

Energy is a natural resource which is used to obtain power for economical development. Chaotic dynamics is necessasarily non-linear dynamics in energy prices

Chaotic behavior is frequently observed in nature, especially in weather systems. Mathematicians have attempted to construct mathematical models to study such behavior. Chaotic systems are initially predictable and then appear to become random.

Crude oil price prediction is a challenging task due to complex non-linear chaotic behavior. Crude oil is a key commodity in global economy. Oil prices are confined between demand and supply. Chaotic systems are predictable for a while and then appear to become random.

The amount of time that the behavior of a chaotic system can be effectively predicted depends on three things: How much uncertainty we tolerate in the forecast, how accurately we can measure its current state, and a time scale depending on the dynamics of the system, called the Lyapunov time .Lapunov exponent is estimated for prices, returns and volatilities.

Crude oil price movement has a chaotic behavior because of large fluctuations in crude oil prices. Oil prices are confined between demand and supply., oil price volatility analysis and oil price forecasting.

Modeling of crude oil prices : As price of oil increases ,quantity demanded decreases.. Price is a decreasing function of demand. As the price increases the supply quantity increases .The demand curve and the supply curve intersect each other at the equilibrium point.

Let P(t) denotes price of crude oil at time t is defined to be the average price at time t
Let D(t) denotes the total amount of demand of crude oil price ,
Q(t) is the quantity demanded of crude oil and $\bar{Q}(t)$ is the quantity stocked of crude oil ,
I(t) is the net quantity imported of crude oil

Assume that dP/dt α ($\bar{Q}(t)$ - Q(t)) then

$$dP/dt = u(\bar{Q}(t) - Q(t)) + rI(t) \quad \text{where } u > 0 , r > 0 \text{, and they are all constants} \qquad (1)$$

where Q(t) is equivalent to quantity of oil stocked Q_0 and volume of gap $S(\tau)$ - $D(\tau)$ i.e. crude oil (τ) supplied and oil demanded.

$$Q(t) \approx Q_0 + \int_0^t (S(\tau) - D(\tau))\, d\tau \qquad (2)$$

Suppose the relation between demand of crude oil and price of crude oil is not linear.

When the price of oil does not cross the limit that purchasing power can bear, the demand of oil stimulates the price of oil and the price is positively correlated with demand.
But when the increasing price of oil crosses the limit , purchasing power can decrease with the increase of en price of oil and both are negatively correlated.

$$D(t) = D_0 - \beta P(t) - \psi\, (P(t).dP(t)/dt \qquad (3)$$

where initial demand $D_0 > 0$, $\beta > 0$, β means marginal demand of oil quantity.

ψ(P(t)) is a function of oil price (P(t))

We consider ψ (P(t)) is a linear function of P(t), then

$$\psi P(t) = bP(t) + c \quad \text{where b and c are constants.} \qquad (4)$$

Supply of oil is oil produced by the enterprise, thus we have

$$S(t) = S_0 + \alpha\, P(t) \quad \text{where initial supply } S_0 \geq 0 , \alpha > 0 \qquad (5)$$

α denotes marginal supply of oil quantity

Suppose Net Import of oil is denoted by I(t) , is a linear function of P(t) , then

$I(t) = (n - m)\, P(t)$ where $m > 0$, $n > 0$ (6)

$d^2P(t) / dt^2 = - u (\alpha + \beta) P(t) - [u (bP(t) + C) + rm] dP(t)/dt + u (D_0 - S_0)$ (7)

Let $P(t) = x(t)$

$dP(t)/ dt = dx(t) /dt = y(t)$

$d^2P(t) / dt^2 = dy /dt$

thus $dx(t) / dt = dP(t) /dt = y(t)$

$dy(t) / dt = d^2P(t) / dt^2 = - u (\alpha + \beta) P(t) - [u (bP(t) + C) + rm] dP(t)/dt + u (D_0 - S_0)$
(8)

Equilibrium Price of oil $= P(t) = (D_0 - S_0 / \alpha + \beta)$

When the demand is more than the supply initially i.e. $D_0 > S_0$, system (8) has the equilibrium point

$(D_0 - S_0 / \alpha + \beta , 0)$ price of oil and demand of oil is equivalent to supply quantity at any time t, so equilibrium price of oil $p\hat{} = D_0 - S_0 / \alpha + \beta$

Let $x(t) = X (t) + P$ and $y(t) = Y(t)$

$dX(t) / dt = Y(t)v$

$dY(t) / dt = - - u (\alpha + \beta) X(t) - [u (bX(t) + p) + c(Y(t) - r m Y(t))$ (9)

This is a plane differential equation system

The Characteristic equation of (9) is

$$\begin{vmatrix} \lambda & -1 \\ u(\alpha + \beta) & \lambda + u\left(b^{\hat{}}\, p + c\right) + r\, m \end{vmatrix} = 0 \quad (10)$$

Simplifying this we get $\lambda^2 + [u(bp\hat{} +c) +r\, m]\lambda + u (\alpha +\beta) = 0$ (11)

Let $\sigma = u (bp\hat{}+ c) +r\, m$ and $\Delta = u(\alpha +\beta)$ then

$\lambda^2 + \sigma\lambda +\Delta = 0$ which is a quadratic equation in λ the characteristic roots are

$$\lambda = \frac{\sigma \pm \sqrt{\sigma 2 - 4\,\Delta}}{2}$$

Let us consider the three possibilities,

i) $\sigma^2 - 4\,\Delta > 0$, then as $\Delta >0$ the two distinct real roots

when $\sigma > 0$, two roots are complex and their real part is negative.

when $\sigma < 0$, two roots are same positive and equilibrium point is stable node

ii) σ^2 - 4 Δ = 0 , then the two roots are positive equal and equilibrium point is instable critical node

iii) σ^2 - 4 Δ < 0 , then the two roots are complex and equilibrium point is asymptotically stable

Equilibrium point of equation (9) is center and stable.

Equilibrium point : As $p^\wedge = D_0 - S_0 / \alpha + \beta$, $\frac{\partial p^\wedge}{\partial D_0} = \frac{1}{\alpha+\beta} > 0$ so the equilibrium price p^ increases with the increase of initial demand D_0. Similarly the relation of equilibrium price p^ with S_0 , α ,β

Now we fix the value of parameter u, α, β , b , D_0 , S_0, m and observe the impact of changing parameter r on equilibrium price.

Let the initial value of r satisfy $\sigma > 0$ and σ^2 - 4 Δ > 0 , the equilibrium price is asymptotically stable node.

If $\sigma > 0$ and σ^2 - 4 Δ = 0 , at this time equilibrium price is asymptotically stable degenerate.

If $\sigma > 0$ and σ^2 - 4 Δ < 0 , at this time equilibrium price is asymptotically stable focus gradually reducing the value of r till $\sigma = 0$ and σ^2 - 4 Δ < 0 equilibrium price is center

At this time equilibrium price has a sign of periodically oscillating, but equilibrium price is stable., gradually reducing the value of r till $\sigma < 0$ and σ^2 - 4 Δ < 0 , equilibrium price is instable. Energy price has a sign of oscillating and being increasing till infinity. Thus we conclude that the impact of changing parameter on equilibrium price.

Conclusion : Crude oil is a key commodity in global economy. Differential equations play a vital role in modeling of crude oil prices. Crude oil price movements has chaotic behavior because of fluctuation in price. We have also seen the stability of system at the equilibrium prices and impact of changing parameters on equilibrium price.

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