THERMAL STRESSES OF A CIRCULAR PLATE WITH INTERNAL HEAT SOURCE: INVERSE PROBLEM

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Abstract – The aim of this paper is to study thermal stresses of a circular plate, in which boundary conditions are of radiation type. We apply integral transform techniques and obtained the solution of the problem. Numerical calculations are carried out for a particular case and results are depicted graphically.

Key Words*:* Thermoelastic response, Circular plate, integral transform, thermal stress, inverse problem.

1. INTRODUCTION

Nowacki [3] has considered the direct and inverse problem of thermo elasticity of a thin circular plate. **Wankhede [6]** has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. **Roy Choudhari [5]** has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero and the fixed circular edge thermally insulated. **Khobragade [7]** has studied Thermoelastic analysis of a thick hollow cylinder with radiation conditions. **Meshram et al. [9]** have discussed steady state thermoelastic problems of semi-infinite hollow cylinder on outer curved surface.

This paper is concerned with inverse transient thermoelastic problem of a circular plate occupying the space $0 \le r \le a,\ -h \le z \le h$ with radiation type boundary conditions.

2. STATEMENT OF THE PROBLEM-I

Consider a circular plate of thickness 2h occupying the space $D : 0 \le r \le a,\quad -h \le z \le h$. The material is isotropic, homogeneous and all properties are assumed to be constant.

The equation for heat conduction as [**3**] is

$$k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\chi(r,z,t)=\frac{\partial T}{\partial t} \tag{2.1}$$

where k is the thermal diffusivity of the material of the plate (which is assumed to be constant).

Subject to the initial and boundary conditions

$$M_t\,(T,1,0,0)=0 \;\; for all\;\; 0\le r\le a,\; -h\le z\le h \tag{2.2}$$

$$M_r\,(T,1,0,a)=G(z,t),(unknown)\; for all\;\; -h\le z\le h,\; t>0 \tag{2.3}$$

$$M_z\,(T,1,k_1,h)=f(r,t)\;\; \text{, for all}\;\; 0\le r\le a,\;\; t>0 \tag{2.4}$$

$$M_z\,(T,1,k_2,-h)=g(r,t)\;\; \text{, for all}\;\; 0\le r\le a,\;\; t>0 \tag{2.5}$$

$$M_r\,(T,1,0,b)=F(z,t),(known)\; for all\;\; -h\le z\le h,\;\; 0\le b\le a,\; t>0 \tag{2.6}$$

The most general expression for these conditions can be given by

$$M_{\nu}\;(f,\bar{k},\bar{\bar{k}},\$)=\left(\bar{k}f+\bar{\bar{k}}\hat{f}\right)_{\nu=\$}$$

where the prime (^) denotes differentiation with respect to ν. $\bar{k}$ and $\bar{\bar{k}}$ are the radiation constant on the upper and lower surface of thin circular plate respectively.

The differential equation governing the displacement function U(r,z,t) as [**2**] is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\nu)\ \alpha_t T \tag{2.7}$$

with $U = 0$ at $r = a$ (2.8)

where ν and a_t are the Poisson ratio and the linear coefficient of thermal expansion of the material of the circular plate.

The stress functions and σ_{rr} *and* $\sigma_{\theta\theta}$ as [**2**] are given by

$$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r} \tag{2.9}$$

$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2} \tag{2.10}$$

where μ is the Lame's constant, while each of the stress functions σ_{rz} , σ_{zz} and $\sigma_{\theta z}$ are zero within the plate in the state of stress.

Equations (2.1) to (2.10) constitute the mathematical formulation of the problem under consideration.

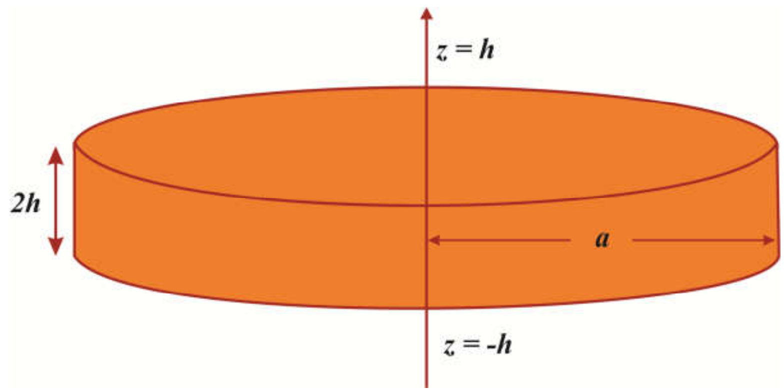


Figure shows the geometry of the problem

3. SOLUTION OF THE PROBLEM

Applying Finite Hankel transform as [4] to the equations (2.5), (2.6), (2.8), (2.9) and using equations (2.7), one obtains

$$k\left[-\xi_n^2\, T^*(\xi_n, z, t) + \frac{\partial^2 T^*(\xi_n, z, t)}{\partial z^2}\right] + \chi^* = \frac{\partial T^*(\xi_n, z, t)}{\partial t} \tag{3.1}$$

where

$$M_t(T^*, 1, 0, 0) = 0 \tag{3.2}$$

$$M_z(T^*, 1, k_1, h) = f^*(\xi_n, t) \tag{3.3}$$

$$M_z(T^*, 1, k_2, -h) = g^*(\xi_n, t) \tag{3.4}$$

$$\chi^* = \int_0^b \chi(r, z, t)\ r\ K_0(\xi_n r)\ dr \tag{3.5}$$

where the symbol (*) means the function in the transformed domain the nucleus for the finite Hankel transform defined by

$$K_0(\xi_n r) = \frac{-\sqrt{2}}{b}\left(\frac{J_0(\xi_n r)}{\xi_n J_0(\xi_n b)}\right).$$

Further applying finite Marchi-Fasulo transform as [1] to the equations (3.1), (3.2) and using (3.3) and (3.4), one obtains

$$k\left[-(\xi_n^2+\mu_m{}^2)\,\overline{T}^*(\xi_n,m,t)+\left[\frac{P(h)f^*}{k_1}-\frac{P(-h)g^*}{k_2}\right]+\chi^*\right]=\frac{d\overline{T}^*(\xi_n,m,t)}{dt} \tag{3.6}$$

$$M_t(\overline{T}^*,1,0,0)=0 \tag{3.7}$$

where $\overline{T}^*$ is transformed function of $\overline{T}$ and m is the transformed parameter. The symbol (-) means a function in the transformed domain and the nucleus is given by the orthogonal function in the internal $-h \le z \le h$ as

$$P_m(z)=Q\,Cos\,(\mu_m z)-W\,Sin\,(\mu_m z)$$

in which

$$Q=\mu_m\,(k_1+k_2)\cos(\mu_m h)$$

$$W=2\cos(\mu_m h)+(k_2-k_1)\,\mu_m\sin(\mu_m h)$$

$$\lambda_m^2=\int_{-h}^{h}P^2(z)\,dz=h\left[Q^2+W^2\right]+Sin\,\frac{(2\mu_m h)}{2\mu_m}\left[Q^2-W^2\right]$$

The eigen values μ_m are the positive roots of the characteristic equation

$$[k_1 a\cos(ah)+\sin(ah)]\;[\cos(ah)+k_2 a\sin(ah)]$$

$$=[k_2 a\cos(ah)-\sin(ah)]\;[\cos(ah)-k_1 a\sin(ah)]$$

After performing calculations on equation (3.6), the reduction is made to linear first order differential equation

$$\frac{d\overline{T}^*(\xi_n,m,t)}{dt}+k(\xi_n^2+\mu_m^2)\,\overline{T}^*(\xi_n,m,t)=\Omega(m,n) \tag{3.7}$$

where

$$\Omega(m,n)=\left[\frac{P_m(h)f^*}{k_1}-\frac{P_m(-h)g^*}{k_2}\right]+\chi^* \tag{3.8}$$

The transformed temperature solution of the differential equation (3.7) is

$$\overline{T}^*=\frac{\Omega(m,n)}{k\left(\xi_n^2+\mu_m{}^2\right)}\left[1-e^{-k(\xi_n^2+\mu_m{}^2)t}\right] \tag{3.9}$$

Applying the inversion theorems of transformation rules defined in (1.1.) and (1.1.), one obtains

$$T(r,z,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\Omega(m,n)}{\lambda_m k\left(\xi_n^2+\mu_m{}^2\right)}\times[1-e^{-k(\xi_n^2+\mu_m{}^2)t}]P_m(z)\,K_0(\xi_n r) \tag{3.10}$$

$$G(z,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\Omega(m,n)}{\lambda_m k\left(\xi_n^2+\mu_m{}^2\right)}\times[1-e^{-k(\xi_n^2+\mu_m{}^2)t}]P_m(z)\,K_0(\xi_n a) \tag{3.11}$$

Equations (3.10) and (3.11) represents the temperature distribution and unknown temperature gradient of a circular plate when there are radiation type boundary conditions.

4. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting value of temperature distribution $T(r,z,t)$ from equation (3.10) in equation (2.10), one obtains the thermoelastic displacement function $U(r,z,t)$ as

$$U(r,z,t) = -(1+v)\, a_t \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\Omega(m,n)}{\xi_n^2 \lambda_m k\left(\xi_n^2+\mu_m{}^2\right)} \times [1-e^{-k(\xi_n^2+\mu_m{}^2)t}] P_m(z)\, K_0(\xi_n r) \tag{4.1}$$

5. DETERMINATION OF STRESS FUNCTION

Using equation (4.1) in equations (2.12) and (2.13), one obtains

$$\sigma_{rr} = (1+v)\, a_t \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\Omega(m,n)}{\xi_n^2 \lambda_m k(\xi_n^2+\mu_m^2)} \times [\,1-e^{-k(\xi_n^2+\mu_m{}^2)t}\,] P_m(z) \times \frac{2\sqrt{2}\mu}{b}\left[\frac{J_1(\xi_n r)}{rJ_0(\xi_n b)}\right] \tag{5.1}$$

$$\sigma_{\theta\theta} = (1+v)\, a_t \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\Omega(m,n)}{\xi_n^2 \lambda_m k(\xi_n^2+\mu_m{}^2)} \times [\,1-e^{-k(\xi_n^2+\mu_m{}^2)t}\,] P_m(z)$$

$$\times \frac{\sqrt{2}}{b}\left[\frac{\xi_n J_0(\xi_n r)}{J_0(\xi_n b)} - \frac{J_1(\xi_n r)}{rJ_0(\xi_n b)}\right] \tag{5.2}$$

6. SPECIAL CASE AND NUMERICAL RESULTS

Set $f(r,t) = r(1-e^{-t})e^{h}$, $g(r,t) = r(1-e^{-t})e^{-h}$, $\chi(r,z,t) = \delta(r-r_0)\delta(z-z_0)\delta(t-t_0)$ (6.1)

Modulus of Elasticity, E (dynes/cm^2)	6.9×10^{11}
Shear modulus, G (dynes/cm^2)	2.7×10^{11}
Poisson ratio, υ	0.281
Thermal expansion coefficient, α_t(cm/cm-^{0}C)	25.5×10^{-6}
Thermal diffusivity, κ (cm^2/sec)	0.86
Thermal conductivity, λ (cal-cm/^{0}C/sec/ cm^2)	0.48
Outer radius, a (cm)	10
Inner radius, b (cm)	9
Thickness, h (cm)	1

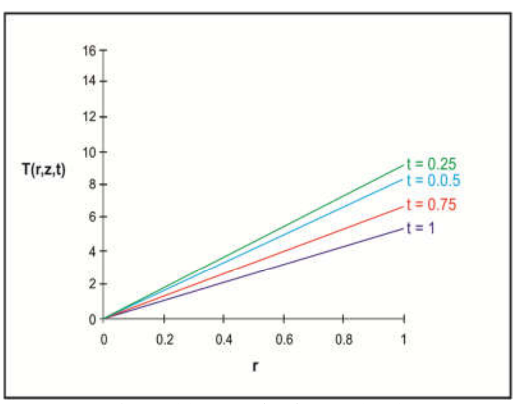


Figure (1) : Graph of r vs T(r,z,t)

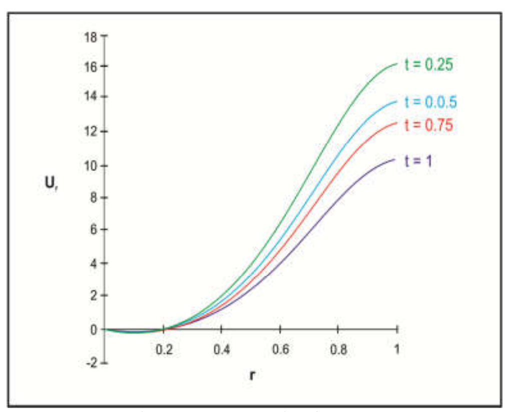


Figure (2) : Graph of r vs u_r

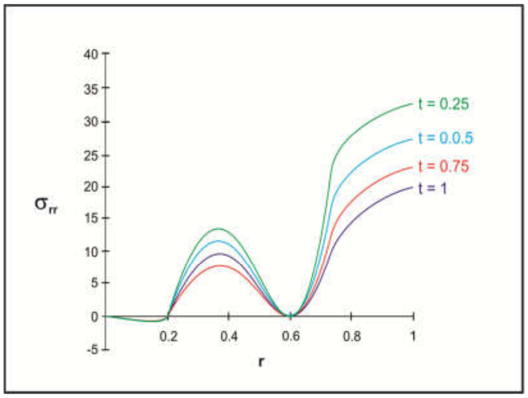


Figure (3) : Graph of r σ_{rr}

7. STATEMENT OF THE PROBLEM-II

Consider a thick circular plate. The material of the plate is isotropic, homogenous and all properties are assumed to be constant. Heat conduction with internal heat source and the prescribed boundary conditions of the radiation type is considered. Equation for heat conduction in cylindrical coordinate as **[3]** is:

$$k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\chi(r,z,t)=\frac{\partial T}{\partial t} \tag{7.1}$$

where k is the thermal diffusivity of the material of the cylinder (which is assumed to be constant), subject to the initial and boundary conditions

$$M_t(T,1,\ 0,0)=0 \text{ , for all } 0\le r\le a\text{ , } -h\le z\le h \tag{7.2}$$

$$M_r(T,1,\ 0,a)=G(z,t)\text{ , (unknown) for all } -h\le z\le h\text{ , } t>0 \tag{7.3}$$

$$M_z(T,1,\ k_1,h)=f(r,t)\quad 0\le r\le a\text{ , } t>0 \tag{7.4}$$

$$M_z(T,1,\ k_2,-h)=g(r,t)\text{ , for all } 0\le r\le a\text{ , } t>0 \tag{7.5}$$

$$M_r(T,1,\ 0,b)=F(z,t)\text{ , (known) for all , } 0\le b\le a\quad t>0 \tag{7.6}$$

The most general expression for these conditions can be given by

$$M_\vartheta(f,\bar{k},\bar{\bar{k}},\$)=(\bar{k}\ f+\bar{\bar{k}}\ \hat{f})_{\vartheta=\$}$$

where the prime (^) denotes differentiation with respect to ϑ; $0\le\eta\le a$; $\bar{k}$ and $\bar{\bar{k}}$ are radiation constants on the upper and lower surfaces of the plate respectively.

The Navier's equations without the body forces for axisymmetric two-dimensional thermoelastic problem can be expressed as **[2]**

$$\nabla^2 u_r-\frac{u_r}{r^2}+\frac{1}{1-2\upsilon}\frac{\partial e}{\partial r}-\frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t\frac{\partial T}{\partial r}=0 \tag{7.7}$$

$$\nabla^2 u_z-\frac{1}{1-2\upsilon}\frac{\partial e}{\partial z}-\frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t\frac{\partial T}{\partial z}=0 \tag{7.8}$$

where u_r and u_z are the displacement components in the radial and axial directions, respectively and the dilatation *e* as

$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \tag{7.9}$$

The displacement function in the cylindrical coordinate system are represented by the Goodier's thermoelastic displacement potential $\phi(r, z, t)$ as **[3]** and Love's function *L* as **[8]**

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z}, \tag{7.10}$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial^2 z} \tag{7.11}$$

in which Goodier's thermoelastic potential as **[3]** must satisfy the equation

$$\nabla^2 \phi = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t T \tag{7.12}$$

and the Love's function *L* as **[8]** must satisfy the equation

$$\nabla^2(\nabla^2 L) = 0 \tag{7.13}$$

Where $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$

The component of the stresses are represented by the use of the potential ϕ and Love's function *L* as **[3]:**

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\upsilon \nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)\right\} \tag{7.14}$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial \phi}{\partial r} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\upsilon \nabla^2 L - \frac{1}{r}\frac{\partial L}{\partial r}\right)\right\} \tag{7.15}$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left((2-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\} \tag{7.16}$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left((1-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\} \tag{7.17}$$

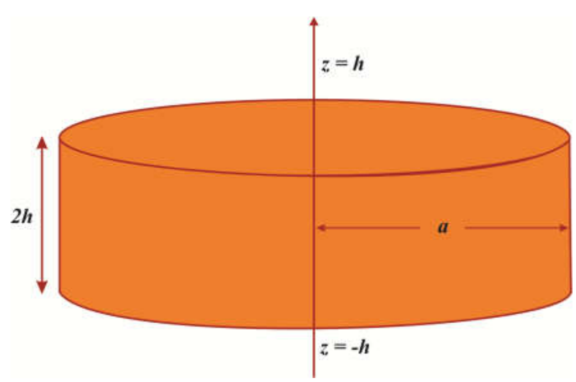


Figure shows the geometry of the problem

where G and υ are the shear modulus and Poisson's ratio respectively.

Equations (7.1) to (7.17) constitute the mathematical formulation of the problem under consideration.

8. SOLUTION OF THE PROBLEM-II

Transient Heat Conduction Analysis:

Applying finite Hankel transform and Marchi-Fasulo transform as [4] , [1] to the equation (7.6) under the condition (7.8)-(7.10), the following reduction is made

$$\frac{d\overline{T}^*}{dt} + k\,\Lambda_{m,n}\,\overline{T}^* = \Omega(\alpha_m, \beta_n) \tag{8.1}$$

where

$$\Lambda_{m,n} = \alpha_m^2 + \beta_n^2$$

and

$$\Omega(\alpha_m, \beta_n) = \left\{ \frac{P_m(h) f^*}{k_1} + \frac{P_m(-h) g^*}{k_2} \right\} + \chi^*$$

in which $K_0(\beta_n r) = \frac{\sqrt{2}}{b}\left(\frac{J_0(\beta_n r)}{J_0(\beta_n b)}\right)$

The eigen values β_n are the positive roots of the characteristic equation $J_0(\beta_n b) = 0,$

Then, the transformed temperature solution of equation (5.3.1) is given by

$$\overline{T}^*(\alpha_m, \beta_n, t) = \frac{\Omega(\alpha_m, \beta_n)}{\kappa\,\Lambda_{m,n}}\,[1 - \exp(-k\,\Lambda_{m,n}\,t)] \tag{8.2}$$

Accomplishing the inversion theorems of transformation rules on equation (8.2), the temperature solution and unknown temperature gradient respectively are shown as follows:

$$T(r,z,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \Pi_{m,n}\,[1 - \exp(-k\,\Lambda_{m,n}\,t)]\ \times P_m(z)\,K_0(\beta_n r) \tag{8.3}$$

$$G(z,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \Pi_{m,n}\,[1 - \exp(-k\,\Lambda_{m,n}\,t)]\ \times P_m(z)\,K_0(\beta_n a)$$

where

$$\Pi_{m,n} = \frac{\Omega(\alpha_m, \beta_n)}{\lambda_m\,(k\,\Lambda_{m,n})}$$

9. THERMOELASTIC DISPLACEMENT

Referring to the fundamental equation (7.1) and its solution (8.3) for the heat conduction problem, the solution for the displacement function is represented by the Goodier's thermoelastic displacement potential ϕ governed by equation (7.12) as

$$\phi(r,z,t) = -\left(\frac{1+\upsilon}{1-\upsilon}\right) a_t \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\Pi_{m,n}}{\Lambda_{m,n}}\,[1 - \exp(-k\,\Lambda_{m,n}\,t)] \times P_m(z)\,K_0(\beta_n r) \tag{9.1}$$

Similarly, the solution for Love's function L as **[8]** are assumed so as to satisfy the governed condition of equation (7.13) as

$$L(r,z,t) = -\left(\frac{1+\upsilon}{1-\upsilon}\right) a_t \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\Pi_{m,n}}{\Lambda_{m,n}} [1-\exp(-k\,\Lambda_{m,n}\,t)]$$

$$\times K_0(\beta_n r) \times [\cosh(\beta_n z) + z\sinh(\beta_n z)] \tag{9.2}$$

In this manner, two displacement functions in the cylindrical coordinate system ϕ and L are fully formulated.

Now, in order to obtain the displacement components, we substitute the values of thermoelastic displacement potential ϕ and Love's function L in equations (7.10) and (7.11), one obtains

$$u_r = -\left(\frac{1+\upsilon}{1-\upsilon}\right) a_t \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\Pi_{m,n}}{\Lambda_{m,n}} [1-\exp(-k\,\Lambda_{m,n}\,t)] \times \left(\frac{\sqrt{2}\,\beta_n J_1(\beta_n r)}{b J_0(\beta_n b)}\right)$$

$$\times \{-Q_m \cos(a_m z) + W_m \sin(a_m z) + z\,\beta_n \cos(\beta_n z)$$

$$+\sin(\beta_n z) + \beta_n \sinh(\beta_n z)\} \tag{9.3}$$

$$u_z = -\left(\frac{1+\upsilon}{1-\upsilon}\right) a_t \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\Pi_{m,n}}{\Lambda_{m,n}} [1-\exp(-k\,\Lambda_{m,n}\,t)] \times K_0(\beta_n r)$$

$$\times \{a_m[Q_m \sin(a_m z) + W_m \cos(a_m z)]$$
$$+ \beta_n[-2+\beta_n + 4\upsilon]\cosh(\beta_n z) + z\,\beta_n^2 \sinh(\beta_n z)\} \tag{9.4}$$

Thus, making use of the two displacement component, the dilation is established as

$$e = -\left(\frac{1+\upsilon}{1-\upsilon}\right) a_t \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\Pi_{m,n}}{\Lambda_{m,n}} [1-\exp(-k\,\Lambda_{m,n}\,t)] \times K_0(\beta_n r)$$

$$\times \{(\beta_n^2 + a_m^2)[Q_m \cos(a_m z) - W_m \sin(a_m z)]$$

$$+ 2(-1+2\upsilon)\beta_n^2 \sinh(\beta_n z)\} \tag{9.5}$$

Then, the stress components can be evaluated by substituting the values of thermoelastic displacement potential ϕ from equation (9.1) and Love's function L from equation (9.2) in equations (7.14) to (7.17), one obtains

$$\sigma_{rr} = 2G\left(\frac{1+\upsilon}{1-\upsilon}\right) a_t \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \left(\frac{\Pi_{m,n}[1-\exp(-k\,\Lambda_{m,n}\,t)]}{\Lambda_{m,n}\sqrt{2}\,b J_0(b\beta_n)}\right)$$

$$\times \{-\beta_n^2 J_2(r\beta_n)(-Q_m \cos(a_m z) + W_m \sin(a_m z) + z\beta_n \cosh(\beta_n z) + (1+\beta_n)\sinh(\beta_n z)$$

$$+ J_0(r\beta_n)[z\beta_n^3 \cosh(\beta_n z) + (2a_m^2 + \beta_n^2)(Q_m \cos(a_m z) - W_m \sin(a_m z))$$

$$+ \beta_n^2 (1+\beta_n + 4\upsilon)\sinh(\beta_n z)]\} \tag{9.6}$$

$$\sigma_{\theta\theta} = 2G\left(\frac{1+\upsilon}{1-\upsilon}\right) a_t \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \left(\frac{\Pi_{m,n}[1-\exp(-k\,\Lambda_{m,n}\,t)]}{\Lambda_{m,n}\sqrt{2}\,b J_0(b\beta_n)}\right)$$

$$\times \{[\beta_n^2 J_2(r\beta_n)[-Q_m \cos(a_m z) + W_m \sin(a_m z) + z\beta_n \cosh(\beta_n z) + (1+\beta_n)\sinh(\beta_n z)]$$

$$+J_0(\beta_n r)[z\beta_n^3\cosh(\beta_n z)+(2a_m^2+\beta_n^2)\times[Q_m\cos(a_m z)-W_m\sin(a_m z)]$$
$$+\beta_n^2(1+\beta_n+4\upsilon)\sinh(\beta_n z)]\} \tag{9.7}$$

$$\sigma_{zz}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)a_t\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{\Pi_{m,n}[1-\exp(-k\Lambda_{m,n}t)]}{\Lambda_{m,n}\sqrt{2}\,bJ_0(b\beta_n)}\right)$$
$$\times\{[\beta_n^2 J_2(r\beta_n)[Q_m\cos(a_m z)-W_m\sin(a_m z)]+J_0(r\beta_n)[-2z\beta_n^3\cosh(\beta_n z)]$$
$$+(2a_m^2+\beta_n^2)[Q_m\cos(a_m z)-W_m\sin(a_m z)]$$
$$-2\beta_n^2(5+\beta_n-4\upsilon)\sinh(\beta_n z)\} \tag{9.8}$$

$$\sigma_{rz}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)a_t\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{\Pi_{m,n}[1-\exp(-k\Lambda_{m,n}t)]}{\Lambda_{m,n}bJ_0(b\beta_n)}\right)$$
$$\times\{\sqrt{2}\,\beta_n J_1(r\beta_n)[a_m W_m\cos(a_m z)+a_m Q_m\sin(a_m z)+\beta_n(\beta_n+2\upsilon)\cosh(\beta_n z)$$
$$+z\beta_n^2\sinh(\beta_n z)]\} \tag{9.9}$$

10. SPECIAL CASE

Set $f(r,t)=r(1-e^{-t})e^{h}$, $g(r,t)=r(1-e^{-t})e^{-h}$, $\chi(r,z,t)=\delta(r-r_0)\delta(z-z_0)\delta(t-t_0)$ (10.1)

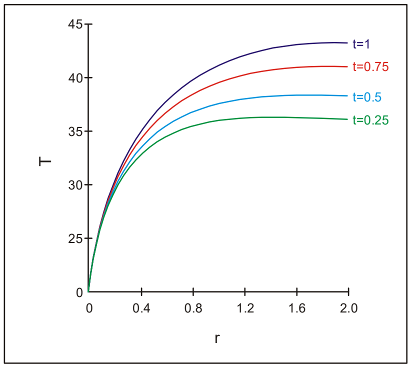
11. NUMERICAL RESULTS, DISCUSSION AND REMARKS

To interpret the numerical computations, we consider material properties of **Aluminum metal**, which can be commonly used in both, wrought and cast forms. The low density of aluminum results in its extensive use in the aerospace industry, and in other transportation fields. Its resistance to corrosion leads to its use in food and chemical handling (cookware, pressure vessels, etc.) and to architectural uses.

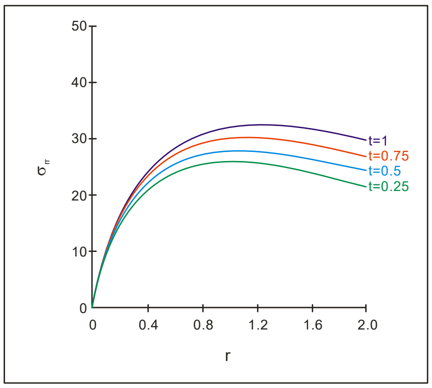
Modulus of Elasticity, E (dynes/cm^2)	6.9×10^{11}
Shear modulus, G (dynes/cm^2)	2.7×10^{11}
Poisson ratio, υ	0.281
Thermal expansion coefficient, α_t(cm/cm-^{0}C)	25.5×10^{-6}
Thermal diffusivity, κ (cm^2/sec)	0.86
Thermal conductivity, λ (cal-cm/^{0}C/sec/ cm^2)	0.48
Outer radius, a (cm)	10
Inner radius, b (cm)	9
Thickness, h (cm)	1.5

Table 1: Material properties and parameters used in this study.
Property values are nominal.

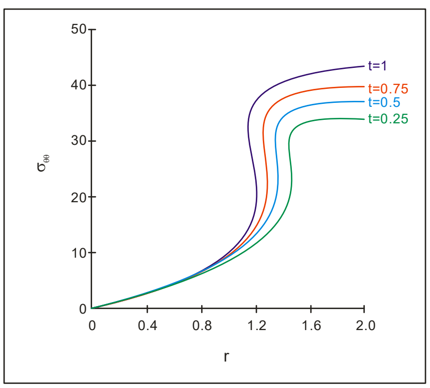
In the foregoing analysis are performed by setting the radiation coefficients constants, $k_1 = 1 = k_2$, so as to obtain considerable mathematical simplicities. The derived numerical results from equation (8.3) to (9.9) have been illustrated graphically.



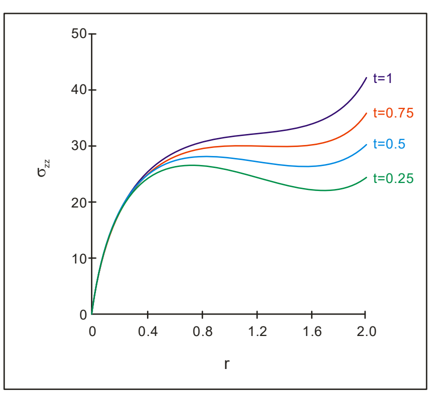
Graph 4: Temperature distribution vs radius



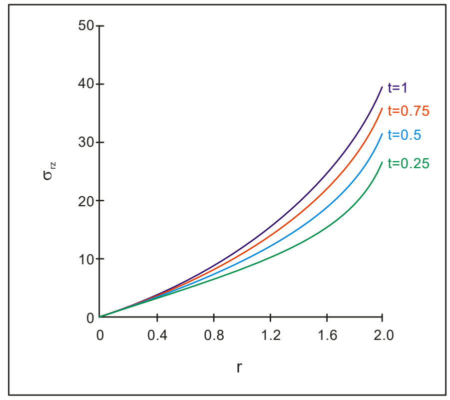
Graph 5: Radial stress distribution vs radius



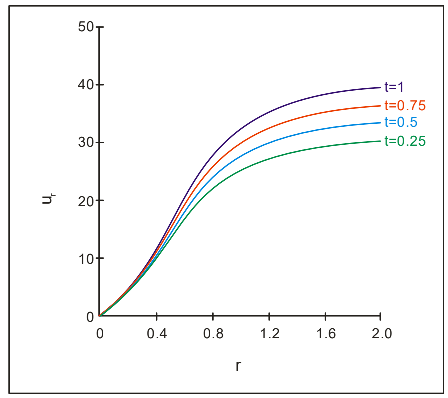
Graph 6: Tangential stress distribution vs radius



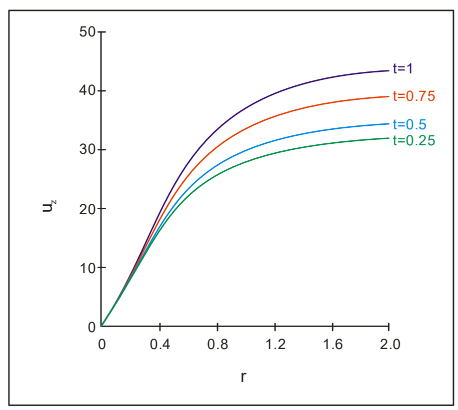
Graph 7: Axial stress distribution vs radius



Graph 8: Shear stress distribution vs radius



Graph 9: Displacement component vs radius



Graph 10: Displacement component vs radius

12. CONCLUSION

In both the problems, the temperature distributions, unknown temperature gradient, displacement and stress functions on $r = a$ of a circular plate have been investigated where internal heat source function is $\delta(r - r_0)\delta(z - z_0)\delta(t - t_0)$. The results have been obtained in terms of Bessel's function in the form of infinite series. The expressions that are obtained can be applied to the design of useful structures or machines in engineering applications. Any particular case of special interest can be assigned to the parameters and functions in the equations.

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