THERMAL STRESSES OF A HOLLOW CYLINDER WITH INTERNAL HEAT SOURCE: INVERSET PROBLEM

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Abstract

This paper is concerned with inverse transient thermoelastic problem , in which we need to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stresses of a finite length hollow circular cylinder with internal heat source . We apply integral transform techniques and obtained the solution of the problem. Numerical calculations are carried out for a particular case and results are depicted graphically.

Key Words*:* Thermoelastic response, Circular cylinder, integral transform, thermal stress, inverse problem

1. INTRODUCTION

Sierakowski and Sun [2] have studied the direct problems of an exact solution to the elastic deformation of finite length hollow cylinder. **Kamdi, et al. [3] have** studied transient thermoelastic problem for a circular solid cylinder with radiation. **Walde** et al. **[4**] have discussed transient thermoelastic problem of a finite length hollow cylinder. **Kulkarni et al. [5]** have derived thermal stresses of a finite length hollow cylinder. **Warbhe et al**. **[6]** discussed numerical study of transient thermoelastic problem of a finite length hollow cylinder. **Lamba et al. [7]** studied analysis of coupled thermal stresses in a axisymmetric hollow cylinder**. Hiranwar et al.** [**8**] studied thermoelastic problem of a cylinder with internal heat sources. **Bagde** et al. [**9**] discussed **heat** conduction problem for a finite elliptic cylinder**.**

Khobragade et al. [10] have investigated **thermal** deflection of a finite length hollow cylinder due to heat generation. **Khobragade [11]** has studied thermoelastic analysis of a thick hollow cylinder with radiation conditions**. Ghume et al. [12]** have derived interior thermo elastic solution of a hollow cylinder. **Chauthale et al. [13]** have studied thermal stress analysis of a thick hollow cylinder. **Singru et al. [14]** have developed integral transform methods for inverse problem of heat conduction with known boundary of semi-infinite hollow cylinder and its stresses. **Fule et al. [15]** have derived thermal stresses of semi-infinite hollow cylinder with internal heat source. **Meshram et al. [15]** have studied steady state thermoelastic problems of semi-infinite hollow cylinder on outer curved surface.

In this paper, an attempt has been made to solve two inverse problems of thermoelasticity. In both the problems, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stresses on outer curved surface of a finite length hollow cylinder.

2. STATEMENT OF THE PROBLEM

Consider a hollow cylinder of length 2h in which sources are generated according to linear function of temperature. The material is isotropic, homogeneous and all properties are assumed to be constant. The equation for heat conduction as [**18**]:

$$k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\chi(r,z,t)=\frac{\partial T}{\partial t} \tag{2.1}$$

where k is the thermal diffusivity of the material of the cylinder (which is assumed to be constant).

Subject to the initial and boundary condition

$$M_t(T,1,0,0)=0 \tag{2.2}$$

$$M_r(T,1,k_1,a)=0 \text{ for all } -h\le z\le h,\ t>0 \tag{2.3}$$

$$M_r(T,1,k_2,\eta)=0 \text{ (known) for all } a\le\eta\le b\ -h\le z\le h,\ t>0 \tag{2.3}$$

$M_r(T,1,0,b) = G(z,t)$ for all $-h \le z \le h,\ t > 0$ (2.4)

$M_z(T,1,k_3,h) = f(r,t)$ (2.5)

$M_z(T,1,k_4,-h) = g(r,t)$, for all $a \le r \le b,\ t > 0$ (2.6)

The most general expression for these conditions can be given by

$$M_v(f,\bar{k},\bar{\bar{k}},\$) = (\bar{k}f + \bar{\bar{k}}\hat{f})_{v=\$}$$

where the prime $(\wedge)$ denotes differentiation with respect to v. $\bar{k}$ and $\bar{\bar{k}}$ are radiation constants on the upper and lower surface of cylinder respectively.

The radiation and axial displacement U and W satisfy the uncoupled thermoelastic equation as **[2]** are

$$\nabla^2 U - \frac{U}{r^2} + (1-2v)^{-1}\frac{\partial e}{\partial r} = 2\left(\frac{1+v}{1-2v}\right)\frac{\partial T}{\partial r} \tag{2.7}$$

$$\nabla^2 W + (1+2v)^{-1}\frac{\partial e}{\partial z} = 2\left(\frac{1+v}{1-v}\right)\alpha_t\frac{\partial T}{\partial z} \tag{2.8}$$

where

$e = \dfrac{\partial U}{\partial r} + \dfrac{U}{r} + \dfrac{\partial W}{\partial z}$ is the volume dilatation,

$$U = \frac{\partial \phi}{\partial r} \tag{2.9}$$

$$W = \frac{\partial \phi}{\partial z} \tag{2.10}$$

The thermoelastic displacement function $\phi(r,z,t)$ as [1] is governed by the Poisson's equation

$$\nabla^2\phi = \left(\frac{1+v}{1-v}\right)\alpha_t T \tag{2.11}$$

with $\phi = 0$ at $r = a$ and $r = b$. (2.12)

where $\nabla^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{\partial^2}{\partial z^2}$,

v and α_t are poisons ratio and the linear coefficient of thermal expansion of the material of the cylinder respectively.

The stress functions are given by

$$\tau_{rz}(a,z,t) = 0,\ \tau_{rz}(b,z,t) = 0,\ \tau_{rz}(r,0,t) = 0 \tag{2.13}$$

$$\sigma_r(a,z,t) = p_1,\ \sigma_r(b,z,t) = -p_0\ \sigma_z(r,0,t) = 0 \tag{2.14}$$

where p_1 and p_0 are the surface pressure assumed to be uniform over the boundaries of the cylinder.

The stress functions are expressed in terms of displacement components by the relations as **[1]:**

$$\sigma_r = (\lambda + 2G)\frac{\partial U}{\partial r} + \lambda\left(\frac{U}{r} + \frac{\partial W}{\partial z}\right) \tag{2.15}$$

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right) \tag{2.16}$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left(\frac{\partial W}{\partial z} + \frac{\partial U}{\partial r}\right) \tag{2.17}$$

$$\tau_{rz} = G\left(\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right) \tag{2.18}$$

where $\lambda = \dfrac{2Gv}{1-2v}$ is the Lame's constant, G is the shear modulus and U and W are the displacement components.

Equations (2.1) to (2.18) constitute the mathematical formulation of the problem under consideration.

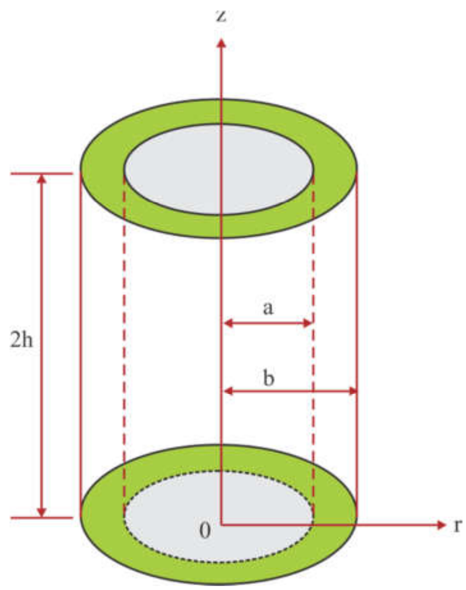


Figure shows the geometry of the problem

3. SOLUTION OF THE PROBLEM

Applying finite Marchi-Zgrablich transform to the equations (2.5), (2.6) and (2.9) and using equation (2.7) and (2.8), one obtains

$$k\left[-\mu_n^2\overline{T}(n,z,t) + \frac{k\,\partial^2\overline{T}(n,z,t)}{\partial z^2}\right] + \chi^* = \frac{\partial\overline{T}(n,z,t)}{\partial t} \tag{3.1}$$

$$M_t(\overline{T}, 1, 0, 0) = 0 \tag{3.2}$$

$$M_z(\overline{T}, 1, k_3, h) = f^*(n,t) \tag{3.3}$$

$$M_z(\overline{T}, 1, k_4, -h) = g^*(n,t) \tag{3.4}$$

where $\overline{T}$ is the transformed function of T and n is the transformed parameter.

The eigen values μ_n are the positive roots of the characteristic equation

$$J_0(k_1, \mu a)\, Y_0(k_2, \mu b) - J_0(k_2, \mu b)\, Y_0(k_1, \mu a) = 0$$

Further applying finite Marchi-Fasulo transform to the equation (3.1) and using (3.3) and (3.4), one obtains

$$k\left[-(\mu_n^2+\xi_m^2)\,\overline{T}^*(n,m,t)+\left[\frac{P_m(h)f^*}{k_3}-\frac{P_m(-h)g^*}{k_4}\right]\right]+\overline{\chi}^* = \frac{d\overline{T}^*(n,m,t)}{dt} \tag{3.5}$$

$$M_t(\overline{T}^*, 1, 0, 0) = 0 \tag{3.6}$$

where $\overline{T}^*$ is the transformed function of $\overline{T}$ and m is the transformed parameter. The symbol (*) means a function in the transform domain and the nucleus is given by the orthogonal functions in the interval $-h \le z \le h$ as

$$P_m(z) = Q_m \cos(\xi_m z) - W_m \sin(\xi_m z)$$

In which

$$Q_m = \xi_m (k_3 + k_4)\cos(\xi_m h)$$

$$W_m = 2\cos(\xi_m h) + (k_3 - k_4)\,\xi_m \sin(\xi_m h)$$

$$\lambda_m = \int_{-h}^{h} p_m^2(z)\,dz = h[Q_m^2 + W_m^2] + \sin\frac{(2\xi_m h)}{2\xi_m}[Q_m{}^2 - W_m^2]$$

The eigen values ξ_m are the positive roots of the characteristic equation

$$[k_3 a\cos(ah) + \sin(ah)][\cos(ah) + k_4 a\sin(ah)]$$

$$= [k_4 a\cos(ah) - \sin(ah)][\cos(ah) - k_3 a\sin(ah)]$$

After performing some calculations on the equation (3.5), the reduction is made to linear first order differential equation as

$$\frac{d\overline{T}^*}{dt} + k(\mu_n^2+\xi_m^2)\,\overline{T}^* = \left\{\left[\frac{P_m(h)f^*}{k_3}-\frac{P_m(-h)g^*}{k_4}\right]+\overline{\chi}^*\right\} \tag{3.6}$$

The transformed temperature solution is

$$\overline{T}^*(n,m,t) = \frac{\Omega(m,n)}{k(\xi_m^2+\mu_n^2)}[1-\exp(-k(\mu_n^2+\xi_m^2)t] \tag{3.7}$$

where

$$\Omega(m,n) = \left\{\left[\frac{P_m(h)f^*}{k_3}-\frac{P_m(-h)g^*}{k_4}\right]+\overline{\chi}^*\right\} \tag{3.8}$$

Applying the inversion of transformation rules, the temperature solution and unknown temperature gradient are obtained as

$$T(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{\lambda_m k(\xi_m^2+\mu_n^2)}\times[1-\exp(-k(\xi_m^2+\mu_n^2)t)]P_m(z)\times S_0(k_1,k_2,\mu_n r) \tag{3.9}$$

$$G(z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{\lambda_m k(\xi_m^2+\mu_n^2)}\times[1-\exp(-k(\xi_m^2+\mu_n^2)t)]P_m(z)\times S_0(k_1,k_2,\mu_n a) \tag{3.10}$$

4. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting value of temperature distribution $T(r,z,t)$ from equation (3.9) in equation (2.11) one obtains the thermoelastic displacement function $\phi(r,z,t)$ as

$$\phi(r,z,t) = -\left(\frac{1+\nu}{1-\nu}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{(\xi_m^2+\mu_n^2)^2\lambda_m k}\right.$$

$$\times\,[1-\exp(-k(\xi_m^2+\mu_n^2)t)]\,P_m(z)]\,\times S_0(k_1,k_2,\mu_n r)\ ] \tag{4.1}$$

Substituting the value of $\phi(r,z,t)$ from equation (4.1) in equations (2.9) and (2.10) one obtains

$$U=-\left(\frac{1+v}{1-v}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{(\xi_m^2+\mu_n^2)^2\lambda_m k}\times[1-\exp(-k(\xi_m^2+\mu_n^2)t)]\,P_m(z)$$

$$\times\mu_n S_0'(k_1,k_2,\mu_n r) \tag{4.2}$$

$$W=-\left(\frac{1+v}{1-v}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{(\xi_m^2+\mu_n^2)^2\lambda_m k}\times[1-\exp(-k(\xi_m^2+\mu_n^2)t)]$$

$$\times(-\xi_m)\,[Q_m\sin(\xi_m z)+W_m\cos(\xi_m z)]\times S_0(k_1,k_2,\mu_n r) \tag{4.3}$$

Making use of two displacement components, the volume dilatation is established as

$$e=-\left(\frac{1+v}{1-v}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{(\xi_m^2+\mu_n^2)^2\lambda_m k}\times[1-\exp(-k(\xi_m^2+\mu_n^2)t)]\,P_m(z)$$

$$\times\left[\mu_n^2 S_0''(k_1,k_2,\mu_n r)+\frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r}-\xi_m^2 S_0(k_1,k_2,\mu_n r)\right] \tag{4.4}$$

5. DETERMINATION OF STRESS FUNCTIONS

The stress components can be evaluated by substituting the values of thermoelastic displacement from equations (4.2) and (4.3) in equations (2.15) to (2.18), one obtains

$$\sigma_r=-\left(\frac{1+v}{1-v}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{(\xi_m^2+\mu_n^2)^2\,\lambda_m\,k}\left[1-\exp(-k(\xi_m^2+\mu_n^2)t)]\,P_m(z)\right]$$

$$\times\left[(\lambda+2G)\,\mu_n S_0''(k_1,k_2,\mu_n r)+\lambda\left(\frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r}-\xi_m^2 S_0(k_1,k_2,\mu_n r)\right)\right] \tag{5.1}$$

$$\sigma_z=-\left(\frac{1+v}{1-v}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{(\xi_m^2+\mu_n^2)^2\,\lambda_m\,k}\ [1-\exp(-k(\xi_m^2+\mu_n^2)t)]\,P_m(z)$$

$$\times\left[(-\lambda+2G)\,\xi_m^2 S_0(k_1,k_2,\mu_n r)+\lambda\left(\mu_n{}^2 S_0''(k_1,k_2,\mu_n r)+\frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r}\right)\right] \tag{5.2}$$

$$\sigma_\theta=-\left(\frac{1+v}{1-v}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{(\xi_m^2+\mu_n^2)^2\,\lambda_m\,k}\,[1-\exp(-k(\xi_m^2+\mu_n^2)t)]\,P_m(z)$$

$$\times\left[(\lambda+2G)\,\frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r}+\lambda(-\xi_m^2 S_0(k_1,k_2,\mu_n r)+\mu_n^2 S_0''(k_1,k_2,\mu_n r)\right] \tag{5.3}$$

$$\tau_{rz}=-2G\left(\frac{1+v}{1-v}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\frac{\Omega(\xi_m,\mu_n)}{(\xi_m^2+\mu_n^2)^2\,\lambda_m k}\,[1-\exp(-k(\xi_m^2+\mu_n^2)t]$$

$$\times[(-\xi_m)\,(Q_m\sin(\xi_m z)+W_m\cos(\xi_m r))\times\mu_n S_0'(k_1,k_2,\mu_n r)] \tag{5.4}$$

6. SPECIAL CASE AND NUMERICAL RESULTS

Set $f(r,t)=re^{h}(1-e^{-t})\,,\,g(r,t)=re^{-h}(1-e^{-t}),\ \chi(r,z,t)=\delta(r-r_0)\delta(z-z_0)\delta(t-t_0)$ (6.1)

Modulus of Elasticity, E (dynes/cm^2)	6.9×10^{11}
Shear modulus, G (dynes/cm^2)	2.7×10^{11}
Poisson ratio, υ	0.281
Thermal expansion coefficient, α_t(cm/cm-^{0}C)	25.5×10^{-6}
Thermal diffusivity, κ (cm^2/sec)	0.86
Thermal conductivity, λ (cal-cm/^{0}C/sec/ cm^2)	0.48
Inner radius, a (cm)	8
Interior radius, c (cm)	9
Outer radius, b (cm)	10
Height, h (cm)	30

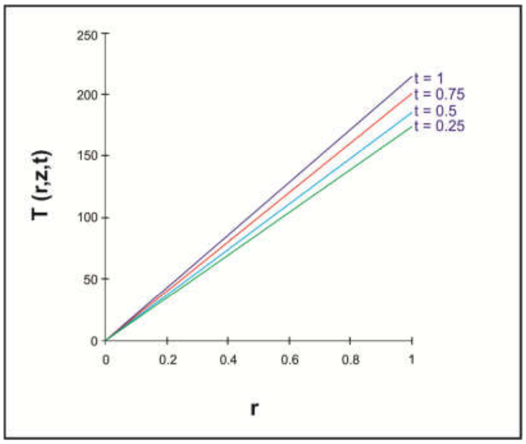
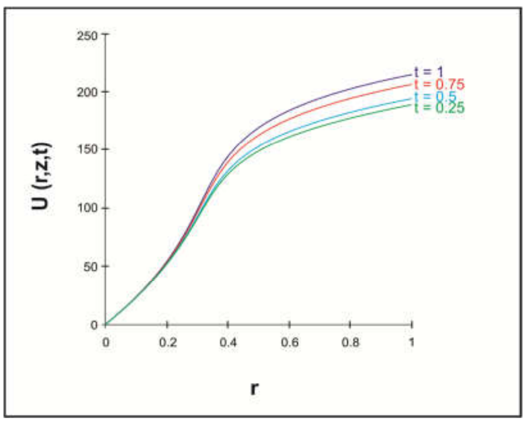
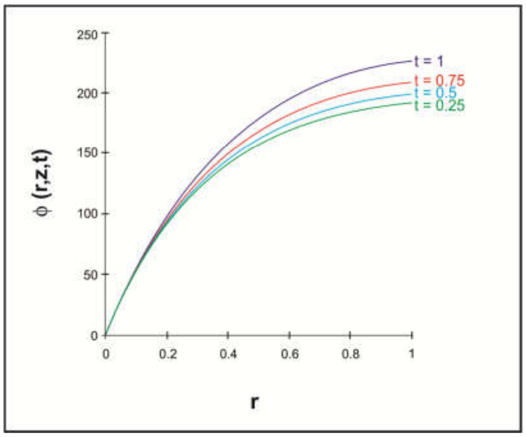


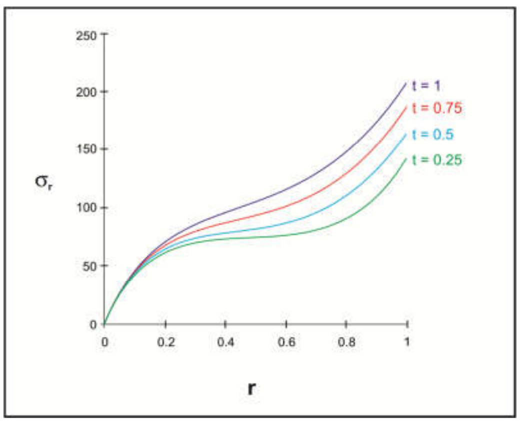
Figure (1). Graph of temperature distribution vs radius



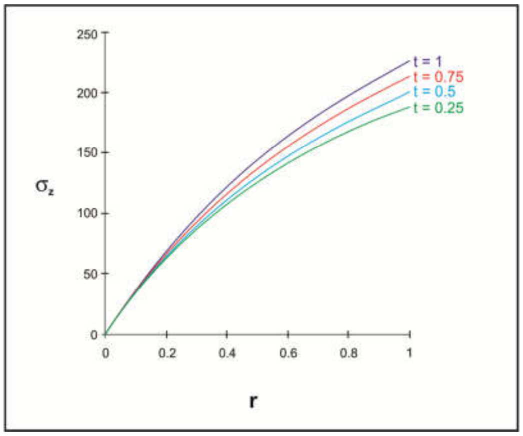
Figure(2). Graph of displacement component vs radius



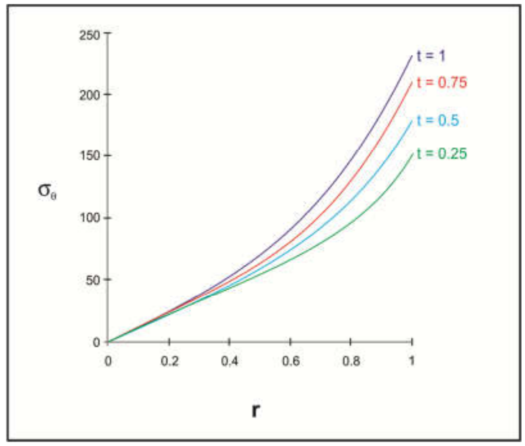
Figure(3). Graph of thermoelastic displacement function vs radius



Figure(4). Graph of radial stress vs radius



Figure(5). Graph of axial stress vs radius



Figure(6). Graph of tangential stress vs radius

7. STATEMENT OF THE PROBLEM-II

Consider a hollow cylinder occupying the space $D=\{(x,y,z)\in R^3 : a\le (x^2+y^2)^{1/2}\le b, -h\le z\le h\}$, where $r=(x^2+y^2)^{1/2}$. The material of the hollow cylinder is isotropic, homogenous and all properties are assumed to be constant. Heat conduction with internal heat source and the prescribed boundary conditions of the radiation type, where the stresses are required to be determined. The equation for heat conduction in cylindrical coordinates as **[18]** is:

$$\kappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\chi(r,z,t)=\frac{\partial T}{\partial t} \tag{7.1}$$

where κ is the thermal diffusivity of the material of the hollow cylinder (which is assumed to be constant), subject to the initial and boundary conditions

$$M_t(T,1,0,0)=T_0, \text{ for all } a\le r\le b, -h\le z\le h \tag{7.2}$$

$$M_r(T,1,k_1,a)=F_1(z,t), \text{ for all } -h\le z\le h, t>0 \tag{7.3}$$

$$M_r(T,1,k_2,c)=F_2(z,t), \text{ (known) for all } -h\le z\le h, t>0 \tag{7.4}$$

$$M_r(T,1,0,b)=G(z,t), \text{ (unknown) for all } -h\le z\le h, t>0 \tag{7.5}$$

$$M_z(T,1,k_3,h)=f(r,t) \text{ for all } a\le r\le b, \quad t>0 \tag{7.6}$$

$$M_z(T,1,k_4,-h)=g(r,t), \text{ for all } a\le r\le b, \quad t>0 \tag{7.7}$$

The most general expression for these conditions can be given by

$$M_\vartheta(f,\bar{k},\bar{\bar{k}},\$)=(\bar{k}\,f+\bar{\bar{k}}\,\hat{f})_{\vartheta=\$}$$

where the prime (^) denotes differentiation with respect to ϑ ; T_0 is the reference temperature; $\bar{k}$ and $\bar{\bar{k}}$ are radiation coefficients respectively.

The Navier's equations without the body forces for axisymmetric two-dimensional thermoelastic problem can be expressed as **[2]**

$$\nabla^2 u_r-\frac{u_r}{r}+\frac{1}{1-2\upsilon}\frac{\partial e}{\partial r}-\frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t\frac{\partial T}{\partial r}=0 \tag{7.8}$$

$$\nabla^2 u_z - \frac{1}{1-2\upsilon}\frac{\partial e}{\partial z} - \frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t \frac{\partial T}{\partial z} = 0 \tag{7.9}$$

where u_r and u_z are the displacement components in the radial and axial directions, respectively and the dilatation e as

$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \tag{7.10}$$

The displacement function in the cylindrical coordinate system are represented by the Goodier's thermoelastic displacement potential $\phi(r, z, t)$ and Love's function L as **[18]**

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z}, \tag{7.11}$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial^2 z} \tag{7.12}$$

in which Goodier's thermoelastic potential must satisfy the equation **[18]**

$$\nabla^2 \phi = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t T \tag{7.13}$$

and the Love's function L must satisfy theequation

$$\nabla^2(\nabla^2 L) = 0 \tag{7.14}$$

where

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$$

The component of the stresses are represented by the use of the potential ϕ and Love's function L as [**18**]

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\upsilon \nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)\right\} \tag{7.15}$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial \phi}{\partial r} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\upsilon \nabla^2 L - \frac{1}{r}\frac{\partial L}{\partial r}\right)\right\} \tag{7.16}$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left((2-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\} \tag{7.17}$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left((1-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\} \tag{7.18}$$

Where$G$andυ are the shear modulus and Poisson's ratio respectively. The boundary condition on the traction free surface stress functions are

$$\sigma_{zz}\big|_{z=\pm h} = \sigma_{rz}\big|_{z=\pm h} = 0 \tag{7.19}$$

Equations (7.1) to (7.19) constitute the mathematical formulation of the problem under consideration.

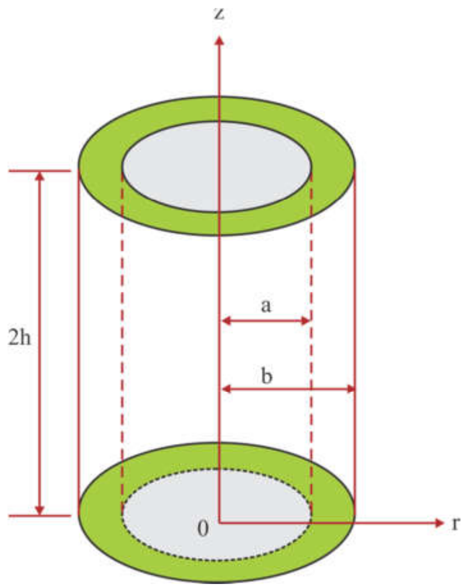


Figure Shows the geometry of the problem

8. SOLUTION OF THE PROBLEM

Transient Heat Conduction Analysis:

Applying finite Marchi-Zgrablich transform to the equations (7.3) to (7.5) and (7.7), and taking into account equations (7.8) and (7.9), one obtains

$$\kappa\left[-\mu_n^2\overline{T}(n,z,t)+\frac{\partial^2\overline{T}(n,z,t)}{\partial z^2}\right]+\overline{\chi}(n,z,t)=\frac{\partial\overline{T}(n,z,t)}{\partial t} \tag{8.1}$$

$$M_t(\overline{T},1,\ 0,0)=\overline{T}_0 \tag{8.2}$$

$$M_z(\overline{T},1,k_3,\ \ h)=\overline{f}(n,t), \tag{8.3}$$

$$M_z(\overline{T},1,\ k_4,-h)=\overline{g}(n,t) \tag{8.4}$$

where $\overline{T}$ is the transformed function of T and n is the transform parameter, and μ_n are the positive roots of the characteristic equation

$$J_0(k_1,\mu a)\ Y_0(k_2,\mu c)-J_0(k_2,\mu c)\ Y_0(k_1,\mu a)=0$$

and F_1, F_2 are assumed to be zero.

Further applying finite Marchi-Fasulo transform to the equations (8.1), (8.2)and using equations (8.3) and (8.4), one obtains

$$\frac{d\overline{T}^*}{dt}+\kappa(\Lambda_{n,m})\overline{T}^*=F(n,\ m) \tag{8.5}$$

where

$$\Lambda_{n,m}=\mu_n^2+a_m^2$$

and

$$F(n,m)=\left\{\frac{P_m(h)\overline{f}}{k_3}-\frac{P_m(-h)\overline{g}}{k_4}+\overline{\chi}^*\right\}$$

Where $\overline{T}^*$ denotes Marchi-Fasulo integral transform of $\overline{T}$ and m is the transform parameter.

The general solution of equation (8.5) is given by

$$\overline{T}^*(n,m,t) = \frac{F(n,m)}{\kappa(\Lambda_{n,m})} + \left[\overline{T}_0^* - \frac{F(n,m)}{\kappa(\Lambda_{n,m})}\right]\exp(-\kappa(\Lambda_{n,m})t) \tag{8.6}$$

Applying inversion theorems of transformation rules to the equation (8.6), one obtain the expression for temperature distribution and unknown temperature gradient as

$$T(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{1}{\lambda_m}[\wp_{n,m} + (\overline{T}_0^* - \wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\} \times P_m(z)\,S_0(k_1,k_2,\mu_n r) \tag{8.7}$$

$$G(z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{1}{\lambda_m}[\wp_{n,m} + (\overline{T}_0^* - \wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\} \times P_m(z)\,S_0(k_1,k_2,\mu_n b) \tag{8.8}$$

where

$$\wp_{n,m} = \frac{F(n,m)}{\kappa(\Lambda_{n,m})}$$

9. THERMOELASTIC SOLUTION

Referring to the fundamental equation (7.1) and its solution (8.7) for the heat conduction problem, the solution for the displacement function are represented by the Goodier's thermoelastic displacement potential ϕ governed by equation (7.13) as

$$\phi(r,z,t) = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \times \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{n,m})}[\wp_{n,m} + (\overline{T}_0^* - \wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]P_m(z)\right\} \times S_0(k_1,k_2,\mu_n r) \tag{9.1}$$

Similarly, the solution for Love's function L are assumed so as to satisfy the governed condition of equation (7.14) as

$$L(r,z,t) = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \times \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{n,m})}[\wp_{n,m} + (\overline{T}_0^* - \wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\}$$

$$\times [B_{nm}\sinh(\mu_n z) + C_{nm} z\cosh(\mu_n z)] \times S_0(k_1,k_2,\mu_n r) \tag{9.2}$$

Using equations (9.1) and (9.2) in equations (7.11) and (7.12), one obtains

$$u_r = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{n,m})}[\wp_{n,m} + (\overline{T}_0^* - \wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\}$$

$$\times \left[\{P_m(z) - [(B_{nm}\mu_n + C_{nm})\cosh(\mu_n z) + C_{nm} z\sinh(\mu_n z)\}\right] \times S_0'(k_1,k_2,\mu_n r) \tag{9.3}$$

$$u_z = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{n,m})}[\wp_{n,m} + (\overline{T}_0^* - \wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\}$$

$$\times \{\, [-a_m(Q_m\sin(a_m z) + W_m\cos(a_m z)) - \mu_n^2(-1+2\upsilon)(B_{nm}\sinh(\mu_n z) + C_{nm} z\cosh(\mu_n z))]$$

$$-2(-1+2\upsilon)C_{nm}\sinh(\mu_n z)\,\mu_n]\; S_0(k_1,k_2,\mu_n r)$$

$$+\mu_n\,(2(1-\upsilon))\,[B_{nm}\sinh(\mu_n z)+C_{nm}\,z\cosh(\mu_n z)]\times[\mu_n\,S_0''(k_1,k_2,\mu_n r)+\frac{S_0'(k_1,k_2,\mu_n r)}{r}\Big\}$$

(9.4)

Thus, making use of the two displacement components, the dilatation can be obtained. Then, the stress components can be evaluated by substituting the values of thermoelastic displacement potential ϕ **[18]** from equation (9.1) and Love's function L from equation (9.2) in equations (7.15) to (7.18), one obtains

$$\sigma_{rr}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{c_n}\times\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{n,m})}\left[\wp_{n,m}+(\overset{*}{T_0}-\wp_{n,m})\exp(-k(\Lambda_{n,m})t)\right]\right\}$$

$$\times\{-P_m(z)[\frac{S_0'(k_1,k_2,\mu_n r)}{r}-a_m^2\,S_0(k_1,k_2,\mu_n r)]$$

$$+\mu_n^2\,(\upsilon-1)S_0''(k_1,k_2,\mu_n r)[\mu_n\left(B_{nm}\cosh(\mu_n z)+C_{nm}\,(z\sinh(\mu_n z)+\cosh(\mu_n z)\right)$$

$$+\mu_n\upsilon[B_{nm}\cosh(\mu_n z)\mu_n+C_{nm}(z\sinh(\mu_n z)+\cosh(\mu_n z))]$$

$$\times[\frac{S_0'(k_1,k_2,\mu_n r)}{r}+\mu_n S_0(k_1,k_2,\mu_n r)]+2\upsilon C_{nm}\,\mu_n^2\cosh(\mu_n z)S_0(k_1,k_2,\mu_n r)]\} \quad (9.5)$$

$$\sigma_{\theta\theta}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\times\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{nm})}[\wp_{n,m}+(\overline{T}_0^*-\wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\}$$

$$\times\{-P_m(z)[\mu_m^2 S_0''(k_1,k_2,\mu_n r)-a_m^2\,S_0(k_1,k_2,\mu_n r)]$$

$$+\frac{\mu_n\,(\upsilon-1)}{r}S_0^1(k_1,k_2,\mu_n r)\ \times[(\mu_n\,B_{nm}+C_{nm})\cosh(\mu_n z)+\mu_n C_{nm}z\sinh(\mu_n z)]$$

$$+\mu_n^2\,\upsilon\,[(\mu_n\,B_{nm}+C_{nm})\cosh(\mu_n z)+\mu_n C_{nm}\,z\sinh(\mu_n z)]$$

$$\times[S_0''(k_1,k_2,\mu_n r)+S_0(k_1,k_2,\mu_n r)]\ +2\upsilon C_{nm}\mu_n^2\cosh(\mu_n z)S_0(k_1,k_2,\mu_n r)]\} \quad (9.6)$$

$$\sigma_{zz}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\times\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{nm})}[\wp_{n,m}+(\overline{T}_0^*-\wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\}$$

$$\times\{-\mu_n P_m(z)[\mu_n S_0''(k_1,k_2,\mu_n r)+\frac{S_0(k_1,k_2,\mu_n r)}{r}]$$

$$+\mu_n^2\,[B_{nm}\cosh(\mu_n z)+C_{nm}\,z\sinh(\mu_n z)](2-\upsilon)$$

$$\times[\mu_n\,S_0''(k_1,k_2,\mu_n r)+r^{-1}\,S_0(k_1,k_2,\mu_n r)]+(1-\upsilon)\,\mu_n\,S_0(k_1,k_2,\mu_n r)$$

$$+\mu_n\,C_{nm}\cosh(\mu_n z)[(2-\upsilon)[\mu_n\,S_0''(k_1,k_2,\mu_n r)+\frac{S_0(k_1,k_2,\mu_n r)}{r}]+(1-\upsilon)\,\mu_n^2\,S_0(k_1,k_2,\mu_n r)] \quad (9.7)$$

$$\sigma_{rz}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\times\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{nm})}[\wp_{n,m}+(\overline{T}_0^*-\wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\}$$

$$\times\{[-\mu_n a_m(Q_m\sin(a_m z)+W_m\cos(a_m z))]\times S_0'(k_1,k_2,\mu_n r)$$

$$+[(B_{nm}\sinh(\mu_n z)+C_{nm}\,z\cosh(\mu_n z))\,\mu_n^2\,[\upsilon\mu_n+\frac{(1-\upsilon)}{r}]-2\upsilon\mu_n^2 C_{nm}\,\sinh(\mu_n z)]\,S_0'(k_1,k_2,\mu_n r)$$

$$+(1-\upsilon)[B_{nm}\sinh(\mu_n z)+C_{nm}\,z\cosh(\mu_n z)]\times[\mu_n^3\,S_0'''(k_1,k_2,\mu_n r)-\frac{S_0(k_1,k_2,\mu_n r)}{r^2}]\} \quad (9.8)$$

10. DETERMINATION OF UNKNOWN ARBITRARY FUNCTION B_{nm} and C_{nm}

Applying boundary conditions (7.7)–(7.11) to the equations (9.1) and (9.2) one obtains

$$B_{nm}=\frac{P_m(h)\bar{X}[h\cosh(\mu_n h)\bar{Y}-\bar{Z}]-a_m\,\bar{f}\,\mu_n\,S_0'(k_1,k_2,\mu_n r)\bar{g}[(2-\upsilon)\bar{X}+(1-\upsilon)\,\mu_n\,S_0(k_1,k_2,\mu_n r)]}{[(2-\upsilon)\bar{X}+(1-\upsilon)\,\mu_n\,S_0(k_1,k_2,\mu_n r)]\{(h\mu_n)\cosh(\mu_n h)[h\cosh(\mu_n h)\bar{Y}-\bar{Z}]-\bar{g}\sinh(\mu_n h)\bar{Y}\}} \quad (10.1)$$

$$C_{nm}=\frac{P_m(h)\bar{X}[\sinh(\mu_n h)\bar{Y}]-a_m\,\bar{f}\,\mu_n\,S_0'(k_1,k_2,\mu_n r)\cosh(\mu_n h)[(2-\upsilon)\bar{X}+(1-\upsilon)\,\mu_n\,S_0(k_1,k_2,\mu_n r)]}{[(2-\upsilon)\bar{X}+(1-\upsilon)\,\mu_n\,S_0(k_1,k_2,\mu_n r)]\{(h\mu_n)\cosh(\mu_n h)[h\cosh(\mu_n h)\bar{Y}-\bar{z}]-\bar{g}\sinh(\mu_n h)\bar{Y}]\}} \quad (10.2)$$

Where $\bar{X}=S_0''(k_1,k_2,\mu_n r)+\dfrac{S_0(k_1,k_2,\mu_n r)}{r}$

$$\bar{Y}=\mu_n^2(\upsilon\mu_n+(1-\upsilon))\frac{S_0'(k_1,k_2,\mu_n r)}{r}+(1-\upsilon)\left(\mu_n^3 S_0'''(k_1,k_2,\mu_n r)-\frac{S_0(k_1,k_2,\mu_n r)}{r^2}\right)$$

$$\bar{f}=W_m\cos(a_m h)+Q_m\sin(a_m h)$$

$$\bar{g}=\cosh(\mu_n h)+(\mu_n h)\sinh(\mu_n h)$$

$$\bar{z}=2\upsilon\mu_n^2\sinh(\mu_n h)\,S_0'(k_1,k_2,\mu_n r)$$

11. SPECIAL CASE

Set $f(r,t)=re^{(h-t)},\ g(r,t)=re^{-(h+t)},\ \chi(r,z,t)=\delta(r-r_0)\delta(z-z_0)\delta(t-t_0)$ (11.1)

12. NUMERICAL RESULTS, DISCUSSION AND REMARKS

To interpret the numerical computations, we consider material properties of Aluminum metal, which can be commonly used in both, wrought and cast forms. The low density of aluminum results in its extensive use in the aerospace industry, and in other transportation fields. Its resistance to corrosion leads to its use in food and chemical handling (cookware, pressure vessels, etc.) and to architectural uses.

Modulus of Elasticity, E (dynes/cm^2)	6.9×10^{11}
Shear modulus, G (dynes/cm^2)	2.7×10^{11}
Poisson ratio, υ	0.281
Thermal expansion coefficient, α_t(cm/cm-^{0}C)	25.5×10^{-6}
Thermal diffusivity, κ (cm^2/sec)	0.86
Thermal conductivity, λ (cal-cm/^{0}C/sec/ cm^2)	0.48
Inner radius, a (cm)	7
Interior radius, c (cm)	9
Outer radius, b (cm)	10
Height, h (cm)	30

Table 1: Material properties and parameters used in this study.

The foregoing analysis are performed by setting the radiation coefficients constants, $k_i = 0.5(i = 1,2)$ and $k_i = 1(i = 3,4)$, so as to obtain considerable mathematical simplicities.

The derived numerical results from equation (8.7) to (9.8) has been illustrated graphically with internal heat source with available additional sectional heat on its flat surface.

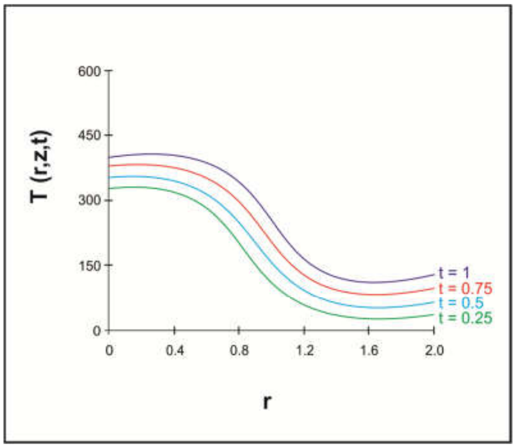


Figure (7). Temperature distribution with internal heat source

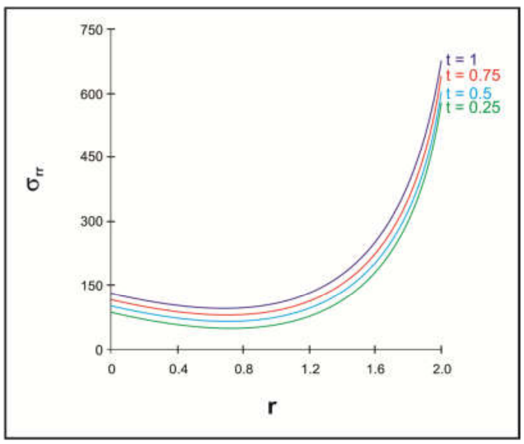


Figure (8). Radial stress distribution with internal heat source

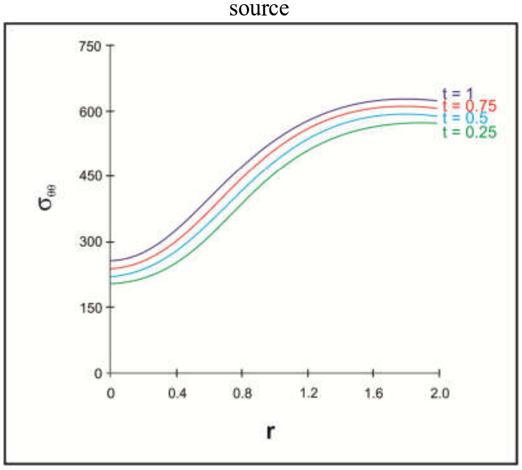


Figure (9). Tangential stress distribution with internal heat source

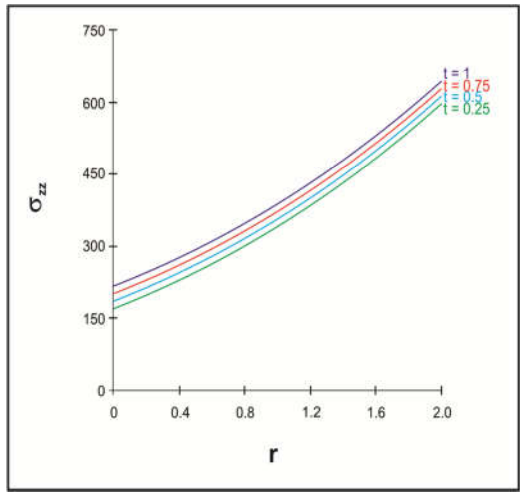


Figure (10). Axial stress distribution for varying along *r*-axis with internal heat source

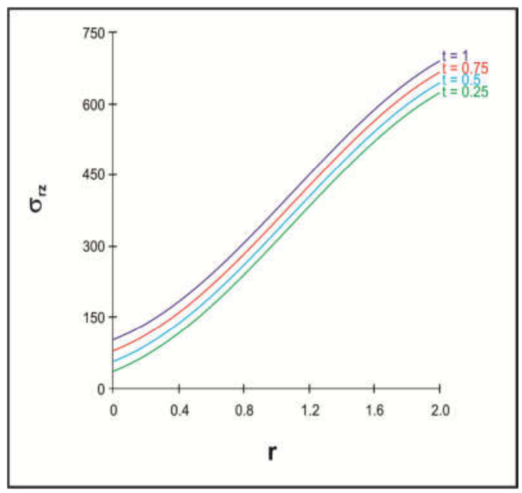


Figure (11). Shear stress distribution for varying along *r*-axis

CONCLUSION

In this study, we treated the two-dimensional thermoelastic problem of a hollow cylinder in which sources are $\chi(r,z,t)=\delta(r-r_0)\delta(z-z_{\cdot 0})\delta(t-t_0)$. We successfully established and obtained the temperature distribution, unknown temperature gradient, displacements and stress functions of the cylinder when the boundary conditions are known. The integral transform techniques are used to obtain the numerical results.The results that are obtainrd can be applied to the design of useful structures or machines in engineering applications. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the applications

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