A New Weibull Rayleigh Distribution with Application to Real Life Data

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Abstract

In this article, we generalize the Weibull Rayleigh distribution using the quadratic rank transmutation map. Various properties of the proposed model such as moments, moment generating functions and order statistics are derived. We discuss the estimation of the model parameters by Maximum Likelihood method and two types of data sets have been considered to make the comparison between the proposed model and its special cases in terms of fitting.

Keywords: Transmuted Weibull Rayleigh distribution, moments, order statistics, reliability and maximum likelihood estimation.

1. Introduction

Since there is numerous data available nowadays that cannot be analyzed using the existing distribution. So, the need of generalizing the existing distributions have been felt by various authors. As a result numerous generalization approaches came into existence to produce the distributions of improved flexibility. Among the numerous generalization techniques available, an important generalization technique is the Quadratic Rank Transmutation Map (QRTM) studied by Shaw and Buckley [1]. A number of authors have considered this generalization approach and successfully attained flexibility over the base model. For instance Merovci [2] studied Transmuted Lindley Distribution , Hussain [3] Studied Transmuted Exponentiated Gamma Distribution: A Generalization of the Exponentiated Gamma Probability Distribution, Aryal and Tsokos [4] studied On the Transmuted Extreme Value Distribution with Application , Dar et al. [5] studied Transmuted Weighted Exponential Distribution and Its Application, Ahmad et al. [6] studied Transmuted Inverse Raleigh Distribution: A Generalization of the Inverse Rayleigh Distribution etc.
The QRTM is given by

$$F_T(x) = (1+\beta)F(x) - \beta\left[F(x)\right]^2, \ |\beta| \le 1 \tag{1.1}$$

where $F(x)$ is the cdf of base distribution.

The main aim of this paper is to generalize the Weibull Rayleigh distribution (WRD) suggested by Aafaq and Ahmad [7] by adding an extra parameter by QRTM. The new distribution is known as Transmuted Weibull Rayleigh Distribution (TWRD).The main motive behind generalization is embedding the existing distribution to more flexible ones. The outline of the paper is as follows: the pdf, cdf and various reliability measures of the new distribution is derived in section 2. Various statistical properties includes moments, skewness, kurtosis, generating function etc. of the TWRD distribution are explored in Section 3. Quantile function, median and random number generation is discussed in section 4. The distribution of the order statistics is expressed in Section 5. In section 6, Maximum likelihood estimates of the parameters of the distribution are discussed .Applications to real data sets is discussed in section 7. Finally, the conclusions is given in section 8.

2. TWRD

The pdf of Weibull Rayleigh distribution is given by

$$f(x) = \frac{\alpha x}{\lambda\theta^2}\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha-1}\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right) \quad ; x>0,\ \alpha,\lambda,\theta>0 . \tag{1.1}$$

The CDF of Weibull Rayleigh distribution is given by

$$F(x) = 1-\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right).$$

(1.2)

Substituting eqn. (1.2) in eqn. (1.1) we get the CDF of TWRD given as

$$F_T(x) = \left\{1-\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}\left\{1+\beta\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\} .$$

(1.3)

The pdf of TWRD is given as

$$f_T(x) = \left\{\frac{\alpha x}{\lambda\theta^2}\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha-1}\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}\left\{1-\beta+2\beta\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)\right)\right\}; x>0,\ \alpha>0,\lambda>0,\theta>0,|\beta|\le 1 \tag{1.4}$$

The graph of PDF for $\alpha = 1.5, \lambda = 1.2, \theta = 1.3$ and different value of the transmuted parameter β is given below

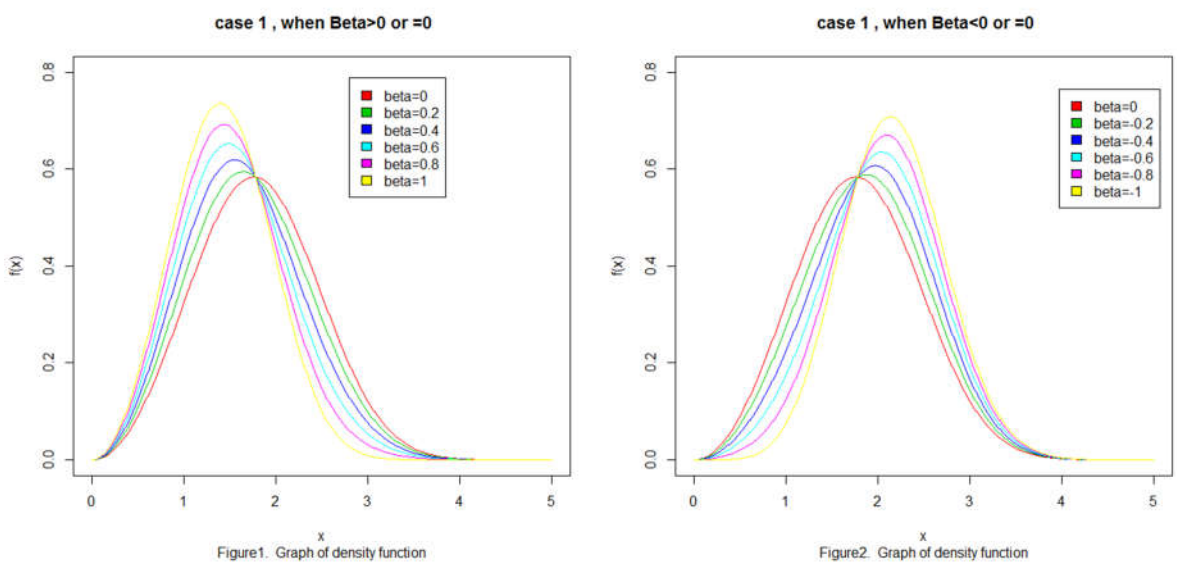

Figure 1: Plots of pdf of TWRD.

2.1. Special cases

For, $\beta = 0$ we obtain WRD given as

$$f(x)=\left\{\frac{\alpha x}{\lambda\theta^2}\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha-1}\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\};x>0,\alpha>0,\lambda>0,\theta>0\ .$$

For, $\beta=0$ $\lambda=1$ and $\alpha=1$ we obtain Rayleigh distribution (RD) given as

$$f(x)=\left\{\frac{x}{\theta^2}\exp\left(-\left(\frac{x^2}{2\theta^2}\right)\right)\right\};x>0,\ \theta>0\ .$$

For, $\lambda=1$ and $\alpha=1$ we obtain Transmuted Rayleigh distribution (TRD) given as

$$f_T(x)=\left\{\frac{x}{\theta^2}\exp\left(-\left(\frac{x^2}{2\theta^2}\right)\right)\right\}\left\{1-\beta+2\beta\exp\left(-\left(\frac{x^2}{2\theta^2}\right)\right)\right\};x>0,\theta>0\left|\beta\right|\le 1$$

2.2. Reliability measures

In this section, reliability measures of TWRD are discussed.

2.2.1. Reliability function

The reliability function of a TWRD distribution is given by

$$R_T(x)=1-\left\{1-\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}\left\{1+\beta\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)\right)\right\}$$

2.2.2. Hazard rate

The hazard rate for a TWRD is given by

$$h_T(x)=\frac{\left\{\frac{\alpha x}{\lambda\theta^2}\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha-1}\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}\left\{1-\beta+2\beta\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)\right)\right\}}{\left[1-\left\{1-\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}\left\{1+\beta\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}\right]}$$

2.2.3. Reverse hazard function

The reverse hazard function of TWRD is given as

$$\phi_T(x)=\frac{\left\{\frac{\alpha x}{\lambda\theta^2}\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha-1}\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}\left\{1-\beta+2\beta\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)\right)\right\}}{\left[\left\{1-\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}\left\{1+\beta\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}\right]}$$

2.2.4. Mean residual life (MRL)

The MRL is given by

$$\mu(t) = E(T - t \mid T > 0) = \frac{1}{R(t)}\begin{cases} \int_t^\infty R(x)\,dx & ; R(t) > 0 \\ = \quad 0 & ; R(t) = 0 \end{cases}$$

MRL for TWRD is given as

$$\mu(t) = \left\{\frac{\alpha t}{\lambda\theta^2}\left(\frac{t^2}{2\lambda\theta^2}\right)^{\alpha-1}\exp\left(-\left(\frac{t^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}^{-1}$$

3. Statistical properties

In this section, various statistical properties of TWRD are discussed.

Theorem 3.1: If a random variable X ~TWRD (λ, θ, α), then the r^{th} moment about origin of TWRD given by

$$\mu_r' = \left(2\lambda\theta^2\right)^{r/2}\Gamma\left(1 + \frac{r}{2\alpha}\right)$$

Proof: The r[th] moment about origin is given by

$$\mu_r' = \int_0^\infty x^r f(x)\,dx$$

Using (1.4) we get

$$\mu_r' = \int_0^\infty x^r\left(\frac{\alpha x}{2\lambda\theta^2}\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha-1}\left\{1 - \beta + 2\beta\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)\right)\right\}\right)dx$$

After simplification we get

$$\mu_r' = \left(2\lambda\theta^2\right)^{r/2}\Gamma\left(1 + \frac{r}{2\alpha}\right)\left[1 - \beta + 2^{-r/2}\beta\right] \qquad (2.1)$$

Hence proved.

3.1. Descriptive Statistics

3.1.1. Moments

The first four moments about origin for TWRD can be respectively obtained by substituting $r = 1,2,3,4$ in equation (2.1) and are given as below:

$$\mu_1' = \left(2\lambda\theta^2\right)^{1/2}\Gamma\left(1 + \frac{1}{2\alpha}\right)\left[1 - \beta + 2^{-1/2}\beta\right].$$

$$\mu_2' = \left(2\lambda\theta^2\right)\Gamma\left(1 + \frac{1}{\alpha}\right)\left[1 - \beta + 2^{-1}\beta\right].$$

$$\mu_3' = \left(2\lambda\theta^2\right)^{3/2}\Gamma\left(1 + \frac{3}{2\alpha}\right)\left[1 - \beta + 2^{-3/2}\beta\right]$$

$$\mu_4' = \left(2\lambda\theta^2\right)^{2}\Gamma\left(1 + \frac{2}{\alpha}\right)\left[1 - \beta + 2^{-2}\beta\right].$$

3.1.2. Mean, Variance, coefficient of variation, Skewness and kurtosis

Mean of TWRD is given by

$$Mean = \left(2\lambda\theta^2\right)^{1/2}\Gamma\left(1+\frac{1}{2\alpha}\right)\left[1-\beta+2^{-1/2}\beta\right].$$

Variance is given by:

$$\sigma^2 = 2\lambda\theta^2\left[\Gamma\left(1+\frac{1}{\alpha}\right)\left[1-\beta+\frac{\beta}{2}\right]-\left[\Gamma\left(1+\frac{1}{2\alpha}\right)\left[1-\beta+2^{-1/2}\beta\right]\right]^2\right].$$

Coefficient of variation is given by:

$$C.V = \sqrt{\frac{\Gamma\left(1+\frac{1}{\alpha}\right)\left[1-\beta+\frac{\beta}{2}\right]}{\left(\Gamma\left(1+\frac{1}{2\alpha}\right)\left[1-\beta+2^{-1/2}\beta\right]\right)^2}-1}\ .$$

Coefficient of skewness is given by:

$$\gamma_1 = \frac{\Gamma\left(1+\frac{3}{2\alpha}\right)\left[1-\beta+2^{-3/2}\beta\right]-3\Gamma\left(1+\frac{1}{\alpha}\right)\left[1-\beta+2^{-1}\beta\right]\Gamma\left(1+\frac{1}{2\alpha}\right)\left[1-\beta+2^{-1/2}\beta\right]+2\left(\Gamma\left(1+\frac{1}{2\alpha}\right)\left[1-\beta+2^{-1/2}\beta\right]\right)^3}{\left(\Gamma\left(1+\frac{1}{\alpha}\right)\left[1-\beta+2^{-1}\beta\right]-\left(\Gamma\left(1+\frac{1}{2\alpha}\right)\left[1-\beta+2^{-1/2}\beta\right]\right)^2\right)^{\frac{3}{2}}}$$

Coefficient of kurtosis is given by:

$$\gamma_2 = \frac{\Gamma\left(1+\frac{2}{\alpha}\right)\left[1-\beta+2^{-2}\beta\right]-4\Gamma\left(1+\frac{1}{2\alpha}\right)\left[1-\beta+2^{-1/2}\beta\right]\Gamma\left(1+\frac{3}{2\alpha}\right)\left[1-\beta+2^{-3/2}\beta\right]+6\Gamma\left(1+\frac{1}{\alpha}\right)\left[1-\beta+2^{-1}\beta\right]\Gamma\left(1+\frac{1}{2\alpha}\right)\left[1-\beta+2^{-1/2}\beta\right]-3\left(\Gamma\left(1+\frac{1}{2\alpha}\right)\left[1-\beta+2^{-1/2\beta}\right]\right)^4}{\left(\Gamma\left(1+\frac{1}{\alpha}\right)\left[1-\beta+2^{-1}\beta\right]-\left(\Gamma\left(1+\frac{1}{2\alpha}\right)\left[1-\beta+2^{-1/2}\beta\right]\right)^2\right)^2}$$

3.1.3. Moment generating function

The mgf. of TWRD is given as

$$M_X(t) = \sum_{r=0}^{\infty}\frac{t^r}{r!}\left(2\lambda\theta^2\right)^{r/2}\Gamma\left(1+\frac{r}{2\alpha}\right)\left[1-\beta+2^{-r/2}\beta\right]$$

4. Quantile function, median and random number generation

Using the inverse cdf method we derive the random numbers for TWRD using the formula

$$\left\{1-\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}\left\{1+\beta\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right\} = u$$

where $u \sim U(0,1)$. After simplification of above equation we get

$$x = \left[2\lambda\theta^2\left[-\log\left[\frac{(\beta-1)-\sqrt{(\beta+1)^2-4\beta u}}{2\beta}\right]\right]^{1/\alpha}\right]^{1/2}$$

The Quantile function is given by

$$x_q = \left[2\lambda\theta^2 \left[-\log\left[\frac{(\beta-1) - \sqrt{(\beta+1)^2 - 4\beta q}}{2\beta} \right] \right]^{1/\alpha} \right]^{1/2}$$

The median is obtained by putting q=0.5 in above eqn.

$$x_{0.5} = \left[2\lambda\theta^2 \left[-\log\left[\frac{(\beta-1) - \sqrt{(\beta+1)^2 - 4\beta(0.5)}}{2\beta} \right] \right]^{1/\alpha} \right]^{1/2}.$$

5. Order Statistics

Let $x_1, x_2, \ldots, x_n$ be an ordered sample of size n from TWRD. Then the PDF of $X_{(r)}$, $X_{(1)}$, $X_{(n)}$, and $X_{(m+1)}$ are respectively given as below:

$$f_{X_{(r):n}}(x) = \begin{cases} \dfrac{n!}{(r-1)!(n-r)!} \left\{ \dfrac{\alpha x}{\lambda\theta^2} \left(\dfrac{x^2}{2\lambda\theta^2} \right)^{\alpha-1} \exp\left(-\left(\dfrac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \left\{ 1 - \beta + 2\beta \exp\left(-\left(\dfrac{x^2}{2\lambda\theta^2} \right) \right) \right\} \\ \left[1 - \left\{ 1 - \exp\left(-\left(\dfrac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \left\{ 1 + \beta \exp\left(-\left(\dfrac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \right]^{n-r} \end{cases}$$

$$f_{X_{1:n}}(x) = n \left\{ \frac{\alpha x}{\lambda\theta^2} \left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha-1} \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \left\{ 1 - \beta + 2\beta \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right) \right) \right\}$$

$$\left[1 - \left\{ 1 - \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \left\{ 1 + \beta \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \right]^{n-1}$$

$$f_{X_{n:n}}(x) = n \left\{ \frac{\alpha x}{\lambda\theta^2} \left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha-1} \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \left\{ 1 - \beta + 2\beta \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right) \right) \right\}$$

$$\left[\left\{ 1 - \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \left\{ 1 + \beta \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \right]^{n-1}$$

$$f_{X_{(m+1):n}}(x) = \frac{(2m+1)!}{m!m!} \left\{ \frac{\alpha x}{\lambda\theta^2} \left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha-1} \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \left\{ 1 - \beta + 2\beta \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right) \right) \right\}$$

$$\left[1 - \left\{ 1 - \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \left\{ 1 + \beta \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \right]^{m}$$

$$\left[\left\{ 1 - \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \left\{ 1 + \beta \exp\left(-\left(\frac{x^2}{2\lambda\theta^2} \right)^{\alpha} \right) \right\} \right]^{m-1}$$

6. Parameter estimation

Let $X_1, X_2, \ldots X_n$ be a random sample of size n from TWRD. Then the log likelihood function of TWRD is given as

$$\log L = n\log\alpha - n\log\lambda - 2n\log\theta + \sum_{i=1}^{n}\log x_i + (\alpha-1)\sum_{i=1}^{n}\log\left(\frac{x^2}{2\lambda\theta^2}\right) - \sum_{i=1}^{n}\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}$$
$$+\sum_{i=1}^{n}\log\left(1-\beta+2\beta\exp\left(-\left(\frac{x^2}{2\lambda\theta^2}\right)^{\alpha}\right)\right)$$

On differentiating log likelihood function with respect to θ, λ and α we get the below given system of nonlinear equations as

$$\frac{\partial}{\partial\theta}\log L = -\frac{2n\alpha}{\theta} - + \frac{2\alpha}{\theta}\sum_{i=1}^{n}\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha} + \frac{4\alpha\beta}{\theta}\sum_{i=1}^{n}\frac{\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\exp\left(-\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\right)}{\left(1-\beta+2\beta\exp\left(-\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\right)\right)}$$

$$\frac{\partial}{\partial\lambda}\log L = \frac{n\alpha}{\theta} - \frac{\alpha}{\lambda}\sum_{i=1}^{n}\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha} + \frac{2\alpha\beta}{\theta}\sum_{i=1}^{n}\frac{\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\exp\left(-\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\right)}{\left(1-\beta+2\beta\exp\left(-\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\right)\right)}$$

$$\frac{\partial}{\partial\alpha}\log L = \frac{n}{\alpha} + \sum_{i=1}^{n}\log\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right) - \sum_{i=1}^{n}\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\log\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right) - 2\beta\sum_{i=1}^{n}\frac{\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\log\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)\exp\left(-\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\right)}{\left(1-\beta+2\beta\exp\left(-\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\right)\right)}$$

$$\frac{\partial}{\partial\beta}\log L = \sum_{i=1}^{n}\frac{2\exp\left(-\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\right)-1}{\left\{1-\beta+2\beta\exp\left(-\left(\frac{x_i^{\,2}}{2\lambda\theta^2}\right)^{\alpha}\right)\right\}}$$

Equating the above system of nonlinear equations respectively to zero and solving them with respect to θ, λ, α and β simultaneously, we obtain the MLE of the respective parameters.

7. Application

In this section we have considered two real life data set to show that TWRD is a better model than its sub models. In order to compare the proposed model with its sub models, the criteria such as -2log$l$, AIC (Akaike information criterion), CAIC (Corrected Akaike information criterion) and HQIC (Hannan-Quinn Information criterion) are considered. The model with least value of -2log l , AIC, AICC and HQIC is considered best fitted.

Data set 1: The first data set represents the remission times (in months) of 124 bladder cancer patients reported by Lee and Wang [8].

Model	MLE's				-log l	AIC	CAIC	HQIC
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\theta}$				
TWRD	0.5629 (0.037)	0.7420 (0.20)	4.049 (488)	5.092 (307)	398.3	804.6	805.0	809.2
WRD	0.5198 (0.034)	-	2.767 (536)	4.0273 (390)	400.4	806.9	807.1	810.4
RD	-	-	-	9.949 (0.446)	477.8	957.6	957.6	958.7
TRD		0.7932 (0.08)	-	11.378 (0.590)	455.1	914.3	914.6	918.8

TABLE 1: MLE's with Standard error in parenthesis and different comparison criteria

Data set 2: The data are from an accelerated life test of 59 conductors, failure times are in hours, and there are no censored observations obtained from [9].

Model	MLE's				-log l	AIC	CAIC	HQIC
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\theta}$				
TWRD	2.540 (0.247)	0.6098 (0.363)	3.118 (179)	3.287 (94)	111.81	231.63	232.06	230.06
WRD	2.349 (0.228)	-	2.007 (97)	3.799 (92)	112.49	230.99	230.61	232.61
RD	-	-	-	5.0637 (0.329)	137.41	276.82	276.89	277.63
TRD		-1.000 (0.5620)	-	4.1802 (0.317)	122.30	248.60	248.81	250.22

Table 2: MLE's with Standard error in parenthesis and different comparison criteria

From Table 1 and Table 2, it can be clearly seen that TWRD has least value of AIC, CAIC and HQIC. Hence TWRD provides the best fit as compared to the other models used for comparison.

8. Conclusion

In this paper the Transmuted Weibull Rayleigh Distribution has been defined and studied. Some basic mathematical properties have been rigorously discussed. The model parameters are estimated by using the Maximum Likelihood estimation. The usefulness of this model is shown by using two real life data sets. We hope that the new model will attract wider application in several areas.

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