INVERSE STEADY-STATE THERMOELASTIC PROBLEMS OF SEMI-INFINITE RECTANGULAR PLATE

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Abstract- This paper is concerned with steady-state thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a semi-infinite rectangular plate when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem

Keywords – Semi-infinite rectangular plate, steady-state problem, Integral transform

1. INTRODUCTION

In 1999, **Adams and Bert [1]** have studied thermoelastic vibrations of a laminated rectangular plate subjected to a thermal shock. **Tanigawa and Komatsubara [2]** have discussed thermal stress analysis of a rectangular plate and its thermal stress intensity factor for compressive stress field. **Vihak et al. [3]** have derived the solution of the plane thermoelasticity problem for a rectangular domain. **Dange et al. [4] have** studied three dimensional inverse transient thermoelastic problem of a thin rectangular plate. **Ghume and Khobragade [5] have** investigated deflection of a thick rectangular plate. **Roy and Khobragade [6] have** discussed transient thermoelastic problem of an infinite rectangular slab. **Lamba and Khobragade [7] have** studied thermoelastic problem of a thin rectangular plate due to partially distributed heat supply.

In 2012**, Sutar and Khobragade [8] have** discussed inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate. **Khobragade et al.** [9] have derived thermal deflection of a thick clamped rectangular plate. **Roy et al. [10] have** studied thermal stresses of a semi infinite rectangular beam. **Jadhav and Khobragade [11] have** discussed inverse thermoelastic problem of a thin finite rectangular plate due to internal heat source. **Singru and Khobragade [12] have** studied thermal stress analysis of a thin rectangular plate with internal heat source. **Further Singru and Khobragade [13]** have **derived**, Thermal stresses of a semi-infinite rectangular slab with internal heat generation.

In this paper, an attempt has been made to solve two inverse steady-state problems of thermoelasticity.

In the first problem, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stresses on the edge $x = a$ of semi-infinite rectangular plate occupying the space D: $0 \le x \le a$, $0 \le y \le \infty$ with the boundary conditions that the heat flux is maintained at zero on the edges $y = 0, \infty$ and temperature is maintained at zero on the edge $x = o$ of semi-infinite rectangular plate.

In the second problem, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stress on the edge $x = a$ of semi-infinite rectangular plate occupying the space D: $0 \le x \le a$, $0 \le y \le \infty$ with the boundary conditions that the heat flux is maintained at zero on the edges $y = 0, \infty$ of semi-infinite rectangular plate and on the edge $x = o$, the temperature is maintained at $h(y)$, which is a known function of y .

2. STATEMENT OF THE PROBLEM-I

Consider semi-infinite rectangular plate occupying the space D : $0 \le x \le a$, $0 \le y \le \infty$. The displacement components u_x and u_y in the x and y- direction represented in the integral form as [2] are

$$u_x = \int \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial x^2} \right) + \alpha T \right] dx \tag{2.1}$$

$$u_y = \int \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2} \right) + \alpha T \right] dy \tag{2.2}$$

where ν and α are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the plate respectively and U(x,y) is the Airy's stress function which satisfy the following relation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 U = -\alpha E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) T \tag{2.3}$$

where E is the Young's modulus of elasticity and T is the temperature of the plate satisfying the differential equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{2.4}$$

subject to the boundary conditions

$$T(0, y) = 0 \tag{2.5}$$

$$T(a, y) = g(y) \text{ (unknown)} \tag{2.6}$$

$$\left[\frac{dT(x,y)}{dy} \right]_{y=0} = 0 \tag{2.7}$$

$$\left[\frac{dT(x,y)}{dy} \right]_{y=\infty} = 0 \tag{2.8}$$

The interior condition is

$$T(\xi, y) = f(y), \; 0 < \xi < a \text{ (known)} \tag{2.9}$$

The stress components in terms of U are given by

$$\sigma_{xx} = \frac{\partial^2 U}{\partial y^2} \tag{2.10}$$

$$\sigma_{yy} = \frac{\partial^2 U}{\partial x^2} \tag{2.11}$$

$$\sigma_{xy} = -\frac{\partial^2 U}{\partial x \partial y} \tag{2.12}$$

Equations (2.1) to (2.12) constitute the mathematical formulation of the problem under consideration.

3. SOLUTION OF THE PROBLEM

Applying Fourier cosine transform to the equations (2.4), (2.5), (2.6) and (2.9) and using the conditions (2.7), (2.8) one obtains

$$\frac{d^2 \overline{T}_C}{dx^2} - p^2 \overline{T}_C = 0 \tag{3.1}$$

where $p^2 = m^2\pi^2$ (3.2)

$$\overline{T}_C(0,m) = 0 \quad (3.3)$$

$$\overline{T}_C(a,m) = \overline{g}_C(m) \quad (3.4)$$

$$\overline{T}_C(\xi,m) = \overline{f}_C(m) \quad (3.5)$$

where $\overline{T}_C$ denotes Fourier cosine transform of T and m is cosine transform parameter.

Equation (3.1) is a second order differential equation whose solution gives

$$\overline{T}_c\ (x,m) = Ae^{px} + Be^{-px} \quad (3.6)$$

where A, B are arbitrary constants.

Using (3.3) and (3.5) in (3.6) one obtains

A + B = 0 (3.7)

$$Ae^{p\xi} + Be^{-p\xi} = \overline{f}_c(m) \quad (3.8)$$

Solving (3.7) and (3.8) one obtains

$$A = \frac{\overline{f}_c(m)}{e^{p\xi} - e^{-p\xi}} \quad , B = -\frac{\overline{f}_c(m)}{e^{p\xi} - e^{-p\xi}}$$

Substituting the values of A and B in (3.6) one obtains

$$\overline{T}_c(x,m) = \overline{f}_c(m)\frac{\sinh(px)}{\sinh(p\xi)} \quad (3.9)$$

Using the condition (3.4) to the solution (3.9) one obtains

$$\overline{g}_c(m) = \overline{f}_c(m)\frac{\sinh(pa)}{\sinh(p\xi)} \quad (3.10)$$

Applying inverse Fourier cosine transform to the equations (3.9)and (3.10) one obtain the expression for temperature distribution T(x,y) and unknown temperature gradient g(y) as

$$T(x,y) = \frac{1}{\pi}\sum_{m=1}^{\infty}\overline{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \quad (3.11)$$

$$g(y) = \frac{1}{\pi}\sum_{m=1}^{\infty}\overline{f}_c(m)\cos py\left[\frac{\sinh(pa)}{\sinh(p\xi)}\right] \quad (3.12)$$

where $\overline{f}_c(m) = \int_0^{\infty} f(y)\cos py\,dy$

Substituting the value of T(x,y) from (3.11) in (2.1) one obtains the expression for Airy's stress function U(x,y) as

$$U(x,y) = -\frac{\alpha E}{\pi p^2}\sum_{m=1}^{\infty}\bar{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \tag{3.13}$$

4. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting the value of U(x,y) from (3.13) in (2.1) and (2.2) one obtains the thermoelastic displacement functions u_x and u_y as

$$u_x = \left[\frac{\alpha(2+\nu)}{\pi p}\right]\sum_{m=1}^{\infty}\bar{f}_C(m)\cos py\left[\frac{\cosh(px)}{\sinh(p\xi)}\right] \tag{4.1}$$

$$u_y = \left[\frac{-\alpha\nu)}{\pi p}\right]\sum_{m=1}^{\infty}\bar{f}_C(m)\sinh(py)\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \tag{4.2}$$

5. DETERMINATION OF STRESS FUNCTIONS

Using (3.13) in (2.10) , (2.11) and (2.12) the stress functions are obtained as

$$\sigma_{xx} = \left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\bar{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \tag{5.1}$$

$$\sigma_{yy} = -\left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\bar{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \tag{5.2}$$

$$\sigma_{xy} = \left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\bar{f}_c(m)\sin py\left[\frac{\cosh(px)}{\sinh(p\xi)}\right] \tag{5.3}$$

6. SPECIAL CASE

Set $f(y) = e^{-y^2}\xi$ (6.1)

Applying finite Fourier cosine transform to the equation (6.1) one obtains

$$\bar{f}_c(m) = \int_0^{\infty} e^{-y^2}\xi\cos(py)dy \qquad = \left(\frac{\xi\sqrt{\pi}e^{-p^2/4}}{2}\right) \tag{6.2}$$

Substituting the value of $\bar{f}_c(m)$ from (6.2) in the equations (3.11) and (3.12) one obtains

$$T(x,y) = \left(\frac{\xi}{2\sqrt{\pi}}\right)\sum_{m=1}^{\infty}e^{-p^2/4}\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \tag{6.3}$$

$$g(y) = \left(\frac{\xi}{2\sqrt{\pi}}\right)\sum_{m=1}^{\infty}e^{-p^2/4}\cos py\left[\frac{\sinh(pa)}{\sinh(p\xi)}\right] \tag{6.4}$$

7. NUMERICAL RESULT

Set $\beta = \dfrac{\xi}{2\sqrt{\pi}}$, π = 3.14, *a* = 2 m, ξ =1.5 m in equation (6.4) to obtain

$$\frac{g(y)}{\beta} = \sum_{m=1}^{\infty} e^{-p^2/4} \cos(1.57my)\left[\frac{\sinh(3.14m)}{\sinh(1.36m)}\right] \tag{7.1}$$

8. STATEMENT OF THE PROBLEM-II

Consider semi-infinite rectangular plate occupying the space D : $0 \le x \le a$, $0 \le y \le \infty$. The displacement components u_x and u_y in the x and y- direction represented in the integral form as [2] are

$$u_x = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial x^2}\right) + \alpha T\right] dx \tag{8.1}$$

$$u_y = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2}\right) + \alpha T\right] dy \tag{8.2}$$

where ν and α are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the plate respectively and U(x,y) is the Airy's stress function which satisfy the following relation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 U = -\alpha E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) T \tag{8.3}$$

where E is the Young's modulus of elasticity and T is the temperature of the plate satisfying the differential equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{8.4}$$

subject to the boundary conditions

$$T(0, y) = h(y) \tag{8.5}$$

$$T(a, y) = g(y) \text{ (unknown)} \tag{8.6}$$

$$\left[\frac{dT(x,y)}{dy}\right]_{y=0} = 0 \tag{8.7}$$

$$\left[\frac{dT(x,y)}{dy}\right]_{y=\infty} = 0 \tag{8.8}$$

The interior condition is

$$T(\xi, y) = f(y) \;,\; 0 < \xi < a \text{ (known)} \tag{8.9}$$

The stress components in terms of U are given by

$$\sigma_{xx} = \frac{\partial^2 U}{\partial y^2} \tag{8.10}$$

$$\sigma_{yy} = \frac{\partial^2 U}{\partial x^2} \tag{8.11}$$

$$\sigma_{xy} = -\frac{\partial^2 U}{\partial x \partial y} \tag{8.12}$$

Equations (8.1) to (8.12) constitute the mathematical formulation of the problem under consideration.

9. SOLUTION OF THE PROBLEM

Applying Fourier cosine transform to the equations (8.4), (8.5) (8.6) and (8.9) and using (8.7), (8.8) one obtains

$$\frac{d^2 \overline{T}c}{dx^2} - p^2 \overline{T}c = 0 \tag{9.1}$$

where $p^2 = m^2 \pi^2$ (9.2)

$$\overline{T}_c(0,m) = \overline{h}_c(m) \tag{9.3}$$

$$\overline{T}_c(a,m) = \overline{g}_c(m) \tag{9.4}$$

$$\overline{T}_c(\xi,m) = \overline{f}_c(m) \tag{9.5}$$

where $\overline{T}_c$ denotes Fourier cosine transform of T and m

is cosine transform parameter.

The equation (9.1) is a second order differential equation whose solution gives

$$\overline{T}_c\ (x,m) = Ae^{px} + Be^{-px} \tag{9.6}$$

where A, B are arbitrary constants.

Using (9.3) and (9.5) in (9.6) one obtains

$$A + B = \overline{h}_c(m) \tag{9.7}$$

$$Ae^{p\xi} + Be^{-p\xi} = \overline{f}_c(m) \tag{9.8}$$

Solving (9.7) and (9.8) one obtains

$$A = \frac{\overline{f}_c(m)}{e^{p\xi} - e^{-p\xi}} - \frac{\overline{h}_c(m)e^{-p\xi}}{e^{p\xi} - e^{-p\xi}} \quad , \quad B = -\frac{\overline{f}_c(m)}{e^{p\xi} - e^{-p\xi}} + \frac{\overline{h}_c(m)e^{p\xi}}{e^{p\xi} - e^{-p\xi}}$$

Substituting the values of A and B in (9.6) one obtains

$$\overline{T}_c(x,m) = \overline{f}_c(m)\frac{\sinh(px)}{\sinh(p\xi)} - \overline{h}_c(m)\frac{\sinh(p(x-\xi))}{\sinh(p\xi)} \tag{9.9}$$

Using the condition (9.4) to the solution (9.9) one obtains

$$\bar{g}_c(m) = \bar{f}_c(m)\frac{\sinh(pa)}{\sinh(p\xi)} - \bar{h}_c(m)\frac{\sinh(p(a-\xi))}{\sinh(p\xi)} \tag{9.10}$$

Applying inverse Fourier cosine transform to the equations (9.9) and (9.10) one obtain the expression for temperature distribution T(x,y) and the unknown temperature gradient g(y) as

$$T(x,y) = \frac{1}{\pi}\sum_{m=1}^{\infty}\bar{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] - \frac{1}{\pi}\sum_{m=1}^{\infty}\bar{h}_c(m)\cos py\left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)}\right] \tag{9.11}$$

$$g(y) = \frac{1}{\pi}\sum_{m=1}^{\infty}\bar{f}_c(m)\cos py\left[\frac{\sinh(pa)}{\sinh(p\xi)}\right] - \frac{1}{\pi}\sum_{m=1}^{\infty}\bar{h}_c(m)\cos py\left[\frac{\sinh(p(a-\xi))}{\sinh(p\xi)}\right] \tag{9.12}$$

where $\bar{f}_c(m) = \int_0^b f(y)\sin py\,dy$,

$$\bar{h}_c(m) = \int_0^b h(y)\sin py\,dy$$

Substituting the value of T(x,y) from (9.11) in (8.3) one obtains the expression for Airy's stress function U(x,y) as

$$U(x,y) = -\frac{\alpha E}{\pi p^2}\sum_{m=1}^{\infty}\bar{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] + \frac{2\alpha E}{\pi p^2}\sum_{m=1}^{\infty}\bar{h}_s(m)\cos py\left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)}\right] \tag{9.13}$$

10. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting the value of U(x,y) from (9.13) in (8.1) and (8.2) one obtains the thermoelastic displacement functions u_x and u_y as

$$u_x = \left[\frac{2\alpha(2+\nu)}{\pi}\right]\sum_{m=1}^{\infty}\bar{f}_c(m)\left[\frac{\sin py}{\sinh(p\xi)}\right]\left[\frac{\cosh(pa)-1}{m}\right] \quad -\left[\frac{2\alpha(2+\nu)}{\pi}\right]\sum_{m=1}^{\infty}\bar{h}_c(m)\left[\frac{\sin py}{\sinh(p\xi)}\right]\left[\frac{\cosh(p(a-\xi))-\cosh(p\xi)}{m}\right] \tag{10.1}$$

$$u_y = \left[\frac{2\alpha(2+\nu)}{\pi}\right]\sum_{m=1}^{\infty}\bar{f}_c(m)\left[\frac{\sinh(px)}{\sinh(p\xi)}\right]\left[\frac{\cos(pa)-1}{m}\right] \quad -\left[\frac{2\alpha(2+\nu)}{\pi}\right]\sum_{m=1}^{\infty}\bar{h}_c(m)\left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)}\right]\left[\frac{\cos(pa)-1}{m}\right] \tag{10.2}$$

11. DETERMINATION OF STRESS FUNCTIONS

Using (9.13) in (8.10), (8.11) and (8.12) the stress functions are obtained as

$$\sigma_{xx} = \left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\bar{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] - \left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\bar{h}_c(m)\cos py\left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)}\right] \tag{11.1}$$

$$\sigma_{yy} = -\left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\bar{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] + \left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\bar{h}_c(m)\cos py\left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)}\right] \tag{11.2}$$

$$\sigma_{xy} = \left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty} \bar{f}_s(m)\cos py\left[\frac{\cosh(px)}{\sinh(p\xi)}\right] - \left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty} \bar{h}_s(m)\cos py\left[\frac{\cosh(p(x-\xi))}{\sinh(p\xi)}\right] \quad (11.3)$$

12. SPECIAL CASE

Set $f(y) = e^{-y^2} e^{\xi}, \quad h(y) = e^{-y^2}$ (12.1)

Applying Fourier cosine transform to the equation (12.1) one obtains

$$\bar{f}_c(m) = \int_0^{\infty} e^{-y^2} e^{\xi} \cos py\, dy = \left(\frac{\sqrt{\pi} e^{-p^2/4} e^{\xi}}{2}\right) \quad (12.2)$$

$$\bar{h}_c(m) = \int_0^{\infty} e^{-y^2} \cos py\, dy = \left(\frac{\sqrt{\pi} e^{-p^2/4}}{2}\right) \quad (12.3)$$

Substituting the values of $\bar{f}_s(m)$ and $\bar{h}_s(m)$ from (12.2) and (12.3) in the equations (9.11) and (9.12) one obtains

$$T(x,y) = \frac{e^{\xi}}{2\sqrt{\pi}} \sum_{m=1}^{\infty} e^{-p^2/4} \cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] - \frac{1}{2\sqrt{\pi}} \sum_{m=1}^{\infty} e^{-p^2/4} \cos py\left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)}\right] \quad (12.4)$$

$$g(y) = \frac{e^{\xi}}{2\sqrt{\pi}} \sum_{m=1}^{\infty} e^{-p^2/4} \cos py\left[\frac{\sinh(pa)}{\sinh(p\xi)}\right] - \frac{1}{2\sqrt{\pi}} \sum_{m=1}^{\infty} e^{-p^2/4} \cos py\left[\frac{\sinh(p(a-\xi))}{\sinh(p\xi)}\right] \quad (12.5)$$

13. NUMERICAL RESULT

Set $\beta = \frac{1}{2\sqrt{\pi}}$, π = 3.14, a = 2 m, ξ =1.5 m in the equation (12.5) to obtain

$$\frac{g(y)}{\beta} = \sum_{m=1}^{\infty} \cos(1.57my)\left\{\left[\frac{\sinh(3.14m)}{\sinh(1.36m)}\right](e^{1.5}) - \left[\frac{\sinh(0.79m)}{\sinh(1.36m)}\right]\right\} \quad (13.1)$$

14 MATERIAL PROPERTIES

The numerical calculations has been carried out for an aluminum (pure) semi-infinite rectangular plate with the material properties as,

Density ρ =169 lb/ft ^{3}

Specific heat = 0.208 Btu/lbOF

Thermal conductivity K = 117Btu/(hr. ftOF)

Thermal diffusivity α = 3.33 ft^2/hr.

Poisson ratio ν = 0.35

Coefficient of linear thermal expansion α_t = 12.84 x 10^{-6}1/F

Lame constant μ = 26.67

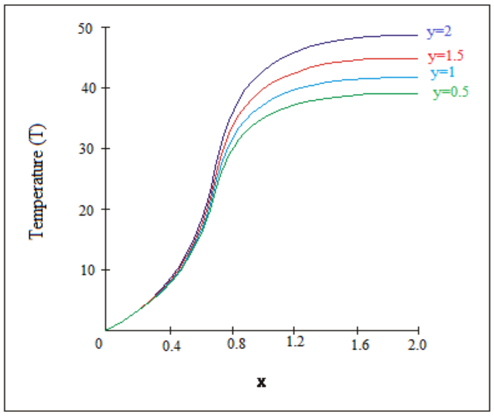
Young's modulus of elasticity E = 70G Pa

15. DIMENSIONS

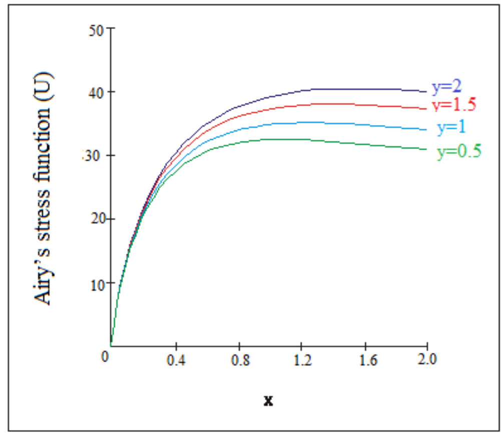
The constants associated with the numerical calculation are taken as

Width of rectangular plate a = 2m

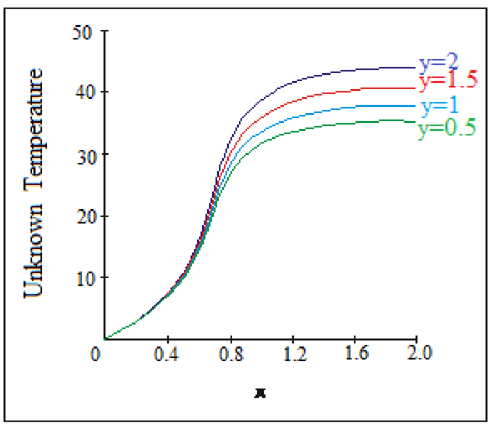
Length of rectangular plate b = 10^2m



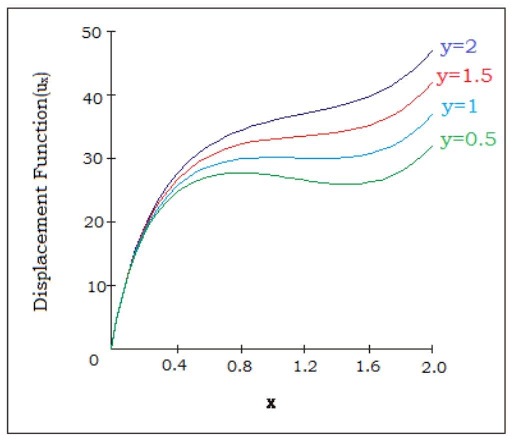
Graph1: Temperature Distribution versus x



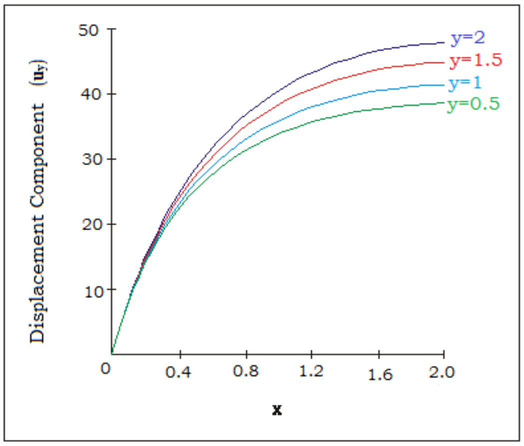
Graph 3: Airy's stress function versus x



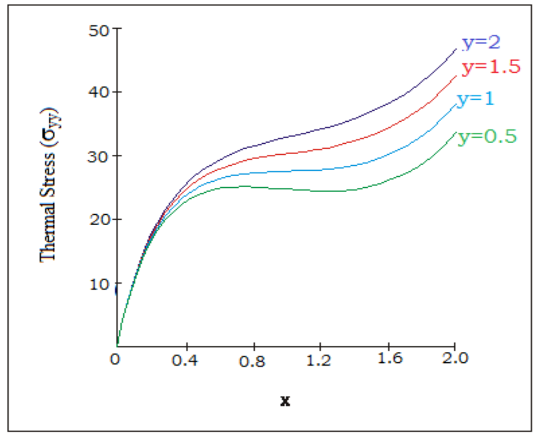
Graph 2: Unknown Temperature versus x



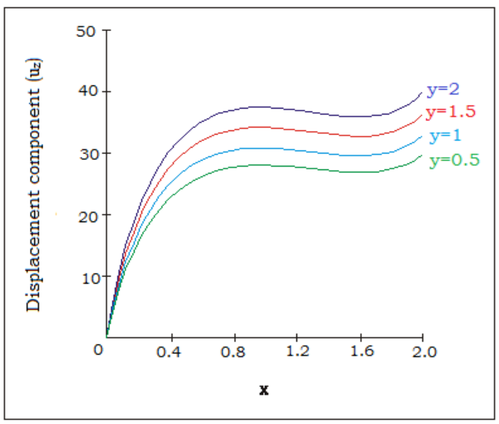
Graph 4: Displacement component versus x



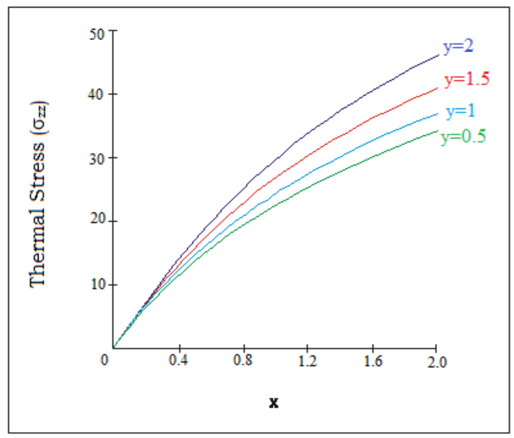
Graph 5: Displacement component versus x



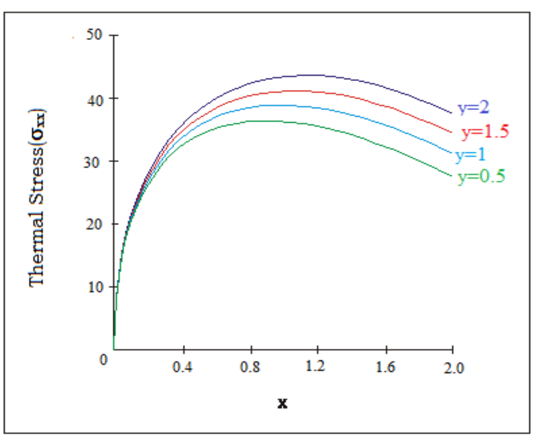
Graph 8: Stress function versus x



Graph 6: Displacement component versus x



Graph 9: Stress function versus x



Graph 7: Stress function versus x

16 DISCUSSION

Graph 1 shows that as the length of rectangular plate (x) increases, the temperature increases gradually for different values of y. Graph 2 shows that as length of rectangular plate (x) increase, unknown temperature also increases and then becomes constant. Graph 3 shows that as length of rectangular plate (x) increase, Airy's stress Function also increases up to certain values of x and then becomes stable.

Graph 4 shows that displacement function (U_x) increase as length of rectangular plate(x) increase at particular value of y. Graph 5 shows that displacement function (U_y) increase as length of rectangular plate (x) increase at particular value of y. Graph 6 shows that displacement function(U_z) increase as length of rectangular plate (x) increase at particular value of y.

Graph 7 shows that thermal stress (σ_{xx})increase as length of rectangular plate (x) increase at particular value of y.

Graph 8 shows that thermal stress (σ_{yy})increase as length of rectangular plate (x) increase at particular value of y.

Graph 9 shows that thermal stress (σ_{zz})increase as length of rectangular plate (x) increase at particular value of y.

17. CONCLUSION

- Two inverse steady state thermoelastic problems of semi-infinite rectangular plate have been discussed.
- The temperature distribution, unknown temperature gradient, displacement and thermal stresses have been investigated with the help of integral transform techniques.
- Infinite series solutions are obtained in the form of bessel's function.
- The solutions that are obtained can be applied to the design of useful structures or machines in engineering applications.
- Numerical calculations have been carried out and depicted graphically.
- Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

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