PARAMETER ESTIMATION OF RC CIRCUITS USING EXTENDED KALMAN FILTER

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Abstract

In this paper, the output voltage estimation of RC low pass filter (LPF) and RC high pass filter (HPF) has been proposed using extended Kalman filter (EKF). The proposed method demonstrates the high signal to noise ratio (SNR) as compared to least mean squares (LMS)method. Also, it has the advantage of easy design and implementation.

Keywords: Extended Kalman filter, parameter estimation, state space representation.

1. INTRODUCTION

Parameter estimation is an important area in the field of science and technology. Various estimation techniques have been used by researcher. There are two basic parameter estimation methods: - 1) methods based on optimization approaches and 2) methods based on stability theory. Differential evolution, particle swarm optimization are examples of optimization based methods and Lyapunov stability method, synchronization approach based on Lasalle's principle are stability based estimation methods. The parameter estimation methods based on the deterministic minimizes error function between model output and measurement data.

Various methods have been used for parameter estimation of systems. In [1], the effects of white noise perturbation on the parameters of electrical network have been analyzed and least square estimation has been used after transferring the deterministic model into stochastic models. [2] presented the total least square estimation of signal parameter via rotation invariance method. [3] proposed anextended stochastic gradient (ESG) filtering and multi innovative filtering for parameter estimation. [4] proposed a parameter estimation of permanent magnet synchronous machine using a dynamic particle swarm optimization method. We used extended Kalman filter for parameter estimation of resistor capacitor (RC) low pass filter (LPF) and high pass filter (HPF). EKF have been used in various applications such as signal processing, communication and navigation control [5]- [9].

RC low pass filter has many applications. It is used as a discrete time repetitive controller for a fly back inverter in continuous conduction mode. The RC low pass filter has been also used for tracking and rejection of periodic signals in a typical frequency range. In [10], RC circuit has application in microelectromechanical systems (MEMS) sensorthat includes a ring oscillator, an RC controlled pulse generator together with a self-tuned inverter converter. The RC high pass filter reduces the bandwidth of noise source. [11] proposed RC low pass filter with good asymptotic behavior in the pass band. In [12], the RC LPF is also used in a flexible continuous time delta sigma modulator. [13] presents RC filter implementation in chopper stabilized thin film transistor low noise amplifier, which is used for EEG signal acquisition and biomarker extraction system. In [14], it is used in linear periodically time varying filter circuit which is used in spectrum scanner.

We proposed the output voltage estimation of RC LPF and HPF using EKF and compared the estimation performance with LMS algorithm. MATLAB simulations show that EKF gives higher SNR as compared to LMS method.

2. STATE SPACE MODELLING OF RC CIRCUITS

A second order dynamic low pass filter (LPF) with R and C components shown in Fig. 1 (a). Let $u_1(t)$ be the input sinusoidal signal. $V_{C_1}(t)$ and $V_{C_2}(t)$ are the capacitor voltages across C_1 and C_2 respectively. The state space model for the circuit shown in Fig.1(a) are obtained by using Kirchhoff current law (KCL). The equations are as follows:

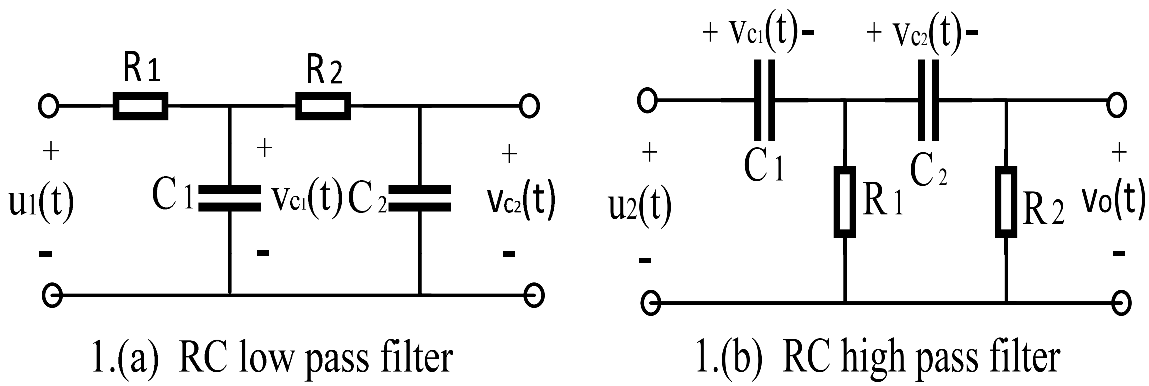


Figure 1. RC Low Pass Filter and High Pass Filter Circuit.

$$\frac{v_{c_1}(t)-u_1(t)}{R_1}+\frac{v_{c_1}(t)-v_{c_2}(t)}{R_2}+C_1\frac{dv_{c_1}}{dt}=0 \tag{1}$$

$$\frac{v_{c_2}(t)-v_{c_1}(t)}{R_2}+C_2\frac{dv_{c_2}}{dt}=0 \tag{2}$$

Representing equations (1) and (2) as state space model, we have

$$\begin{bmatrix} \frac{dv_{c_1}}{dt} \\ \frac{dv_{c_2}}{dt} \end{bmatrix}=\begin{bmatrix} \left(-\frac{1}{R_1C_1}-\frac{1}{R_2C_1}\right) & \frac{1}{R_1C_1} \\ \frac{1}{R_2C_2} & -\frac{1}{R_2C_2} \end{bmatrix}\begin{bmatrix} v_{c_1}(t) \\ v_{c_2}(t) \end{bmatrix}+\begin{bmatrix} \frac{1}{R_1C_1} \\ 0 \end{bmatrix}u_1(t) \tag{3}$$

The output voltage of RCLPF can be written as:

$$y = v_{c_2}(t) \tag{4}$$

Or representing (3) and (4) in state space model, we have

$$\frac{dx}{dt} = Ax + Bu \tag{5}$$

$$y = Cx + Du \tag{6}$$

x represents the state vector and u is input vector at the time k. A , B and C are:

$$A = \begin{bmatrix} \left(-\frac{1}{R_1C_1} - \frac{1}{R_2C_1}\right) & \frac{1}{R_1C_1} \\ \frac{1}{R_2C_2} & -\frac{1}{R_2C_2} \end{bmatrix},\ B = \begin{bmatrix} \frac{1}{R_1C_1} & 0 \end{bmatrix}^T,\ C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

The state model in (5) and (6) can be discretized using first order exponential method. The transformed equations are expressed as

$$x_{k+1} = A_k x_k + B_k u_k + w_k \tag{7}$$

$$y_k = C_k x_k + D_k u_k + v_k \tag{8}$$

(7) and (8) are discrete representation of equations (5) and (6) respectively obtained using $t = k\Delta T$, where $k = 1, 2, 3...$ and ΔT is the sampling time. The matrices A_k , B_k , C_k and D_k are obtained by discretizing A, B, C and D matrices respectively. They are

$$A_k = e^{\Delta T} = I + A\Delta T = \begin{bmatrix} 1-\left(\frac{\Delta T}{R_1C_1} + \frac{\Delta T}{R_2C_1}\right) & \frac{\Delta T}{R_1C_1} \\ \frac{\Delta T}{R_2C_2} & 1-\frac{\Delta T}{R_2C_2} \end{bmatrix} B_k = B\Delta T = \begin{bmatrix} \frac{\Delta T}{R_1C_1} & 0 \end{bmatrix}^T,$$

$$C_k = C = \begin{bmatrix} 0 & 1 \end{bmatrix} \text{ and } D_k = \begin{bmatrix} 0 \end{bmatrix}.$$

Similarly, state space model for HPF shown in Fig 1(b) can be expressed as

$$\begin{bmatrix} \frac{dv_{c_1}}{dt} \\ \frac{dv_{c_2}}{dt} \end{bmatrix} = \begin{bmatrix} \left(-\frac{1}{R_1C_1} - \frac{1}{R_2C_1}\right) & -\frac{1}{R_2C_1} \\ -\frac{1}{R_2C_2} & -\frac{1}{R_2C_2} \end{bmatrix} \begin{bmatrix} v_{c_1}(t) \\ v_{c_2}(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1C_1} + \frac{1}{R_2C_1} \\ \frac{1}{R_2C_2} \end{bmatrix} u_2\ (t) \tag{9}$$

$$y = [-1 \quad -1]\begin{bmatrix} v_{c_1}(t) \\ v_{c_2}(t) \end{bmatrix} + u_2(t) \tag{10}$$

where the matrices A_k , B_k , C_k and D_k for RC HPF can be obtained by discretizing equations (9)-(10) i.e. by substituting $t = k\Delta T$. For HPF circuit, they are:

$$A_k = \begin{bmatrix} 1 - \left(\frac{\Delta T}{R_1 C_1} + \frac{\Delta T}{R_2 C_1}\right) & -\frac{\Delta T}{R_2 C_1} \\ -\frac{\Delta T}{R_2 C_2} & 1 - \frac{\Delta T}{R_2 C_2} \end{bmatrix}, B_k = \begin{bmatrix} \left(\frac{\Delta T}{R_1 C_1} + \frac{\Delta T}{R_2 C_1}\right) & \frac{\Delta T}{R_2 C_2} \end{bmatrix}^T,$$

$$C_k = \begin{bmatrix} -1 & -1 \end{bmatrix} \text{ and } D_k = [1].$$

3. EXTENDED KALMAN FILTER ALGORITHM

EKF is broadly used as state estimation method. It consists of nonlinear state model and observation model. EKF is the best linear estimator with respect to minimum-mean-squared error. The EKF repeatedly computes set of recursive equations as the system operates. In general, a nonlinear system can be mathematically represented as

$$x_k = f(x_{k-1}, u_k) + w_{k-1} \tag{11}$$

$$y_k = g(x_k) + v_k \tag{12}$$

f and g are the nonlinear functions of process and observation model x_k and y_k respectively. u_k is the input vector at the time k . w_k and v_k are the white Gaussian noise with zero mean and variance $Q_k = E[w_k w_k^T]$ and $R_k = E[v_k v_k^T]$ respectively. Expanding (11) and (12) using Taylor's series, we have

$$x_k \equiv f(\hat{x}_{k-1|k-1}) + J_k(x_{k-1} - \hat{x}_{k-1|k-1}) + higher\,order\,terms \tag{13}$$

$$y_k \equiv g(\hat{x}_{k-1|k-1}) + H_k(x_{k-1} - \hat{x}_{k-1|k-1}) + higher\,order\,terms \tag{14}$$

J_k and H_k are Jacobian matrices that give partial derivates of f and g with respect to x .Therefore, we have

$$J_k = \frac{d}{dx_{k-1}} f(\hat{x}_{k-1|k-1}) \text{ and } H_k = \frac{d}{dx_k} g(x_k) \tag{15}$$

The higher order terms can be eliminated for simplify (13) and (14). The EKF steps are as follows:

3.1 Initialization of state

This step sets previous state $\hat{x}_{k-1|k-1}$ with covariance $P_{k-1|k-1}$. This step also initializes Q_k and R_k .

3.2 Prediction of state

After obtaining J_k using (15), predicted state is computed using A_k and B_k as follows:

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1})$$

or

$$\hat{x}_{k|k-1} = A_k \hat{x}_{k-1|k-1} + B_k u_k \tag{16}$$

Predicted covariance

$$P_{k|k-1} = A_k P_{k-1|k-1} A_k^{\ T} + Q_k \tag{17}$$

3.3 State update

This step obtains matrices C_k .

Kalman gain: This step computes the Kalman gain (K_K) using the following equation

$$K_K = P_{k|k-1} \mathrm{C_k}^T (\mathrm{C_k} P_{k|k-1} \mathrm{C_k}^T + R_k)^{-1} \tag{18}$$

Estimation of state and error covariance: This step update estimation with measurements as

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (Y_K - \mathrm{C_k}(\hat{x}_{k|k-1}) \tag{19}$$

And updates error covariance as

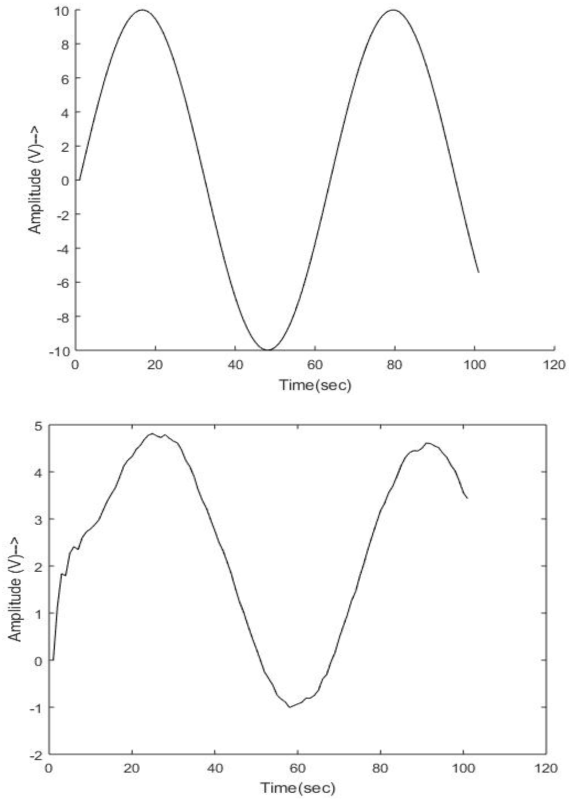
$$P_{k|k} = P_{k|k-1}(\mathrm{I} - K_k \mathrm{C_k}) \tag{21}$$

Repeat step 3.2.

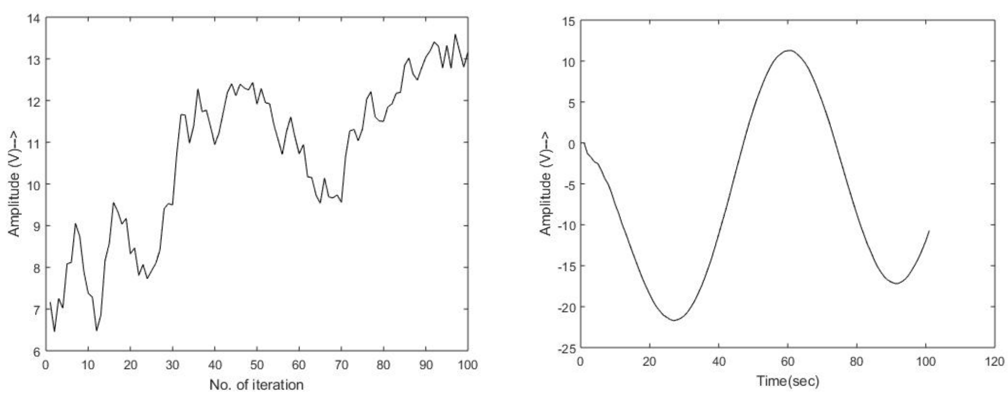
After initialization, EKF uses time update and measurement updates.

4. SIMULATION RESULTS

We estimated the output voltage of RC LPF and RC HPF for two different inputs. Fig. 2(a) shows the applied sinusoidal input with maximum amplitude of 10 V and frequency 0.01 Hz. The white Gaussian noise of zero mean and 0.5 variance has been used for estimation purpose. The output response of LPF circuit using EKF and LMS is shown in Fig 2 (b) and Fig 2 (c) respectively. Fig 2 (d) and Fig. 2 (e) show output voltage estimation of HPF using EKF and LMS respectively for sinusoidal noisy input. Fig. 2(f) and (g) shows the LPF estimated output voltage for noisy square wave input signal. Similarly, Fig. 2(h) and (i) shows output voltage estimation of HPF for noisy square wave input signal using EKF and LMS respectively. Table I shows the SNR in each case.


Amplitude (V)-->
Time(sec)
Amplitude (V)-->
Time(sec)

(a) (b)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No. of iteration
Amplitude (V)-->
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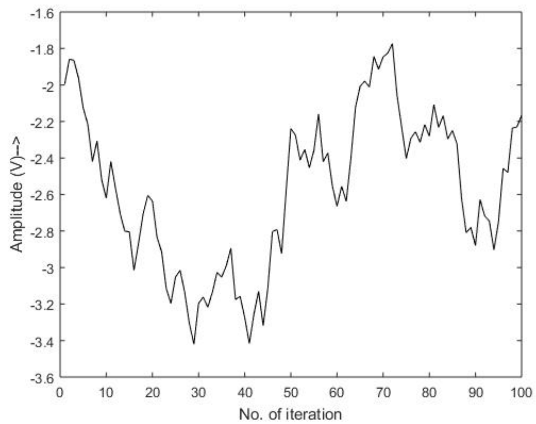
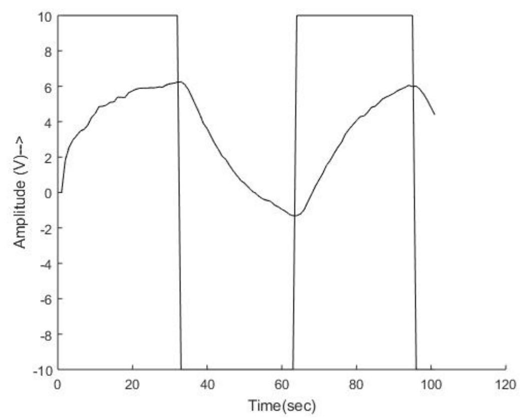
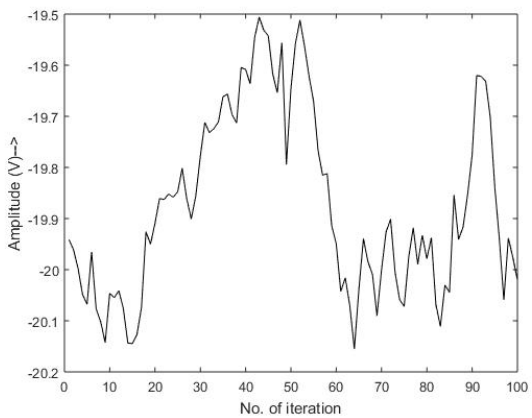
(c) (d)



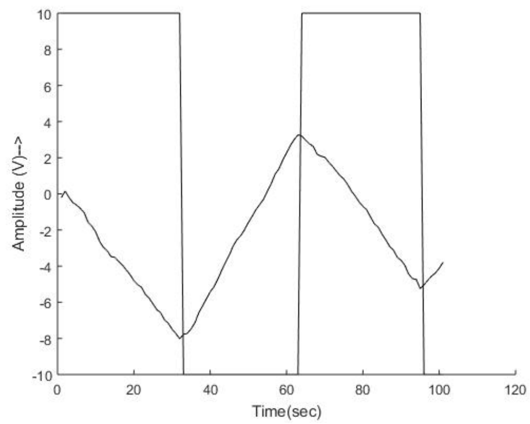
(e)



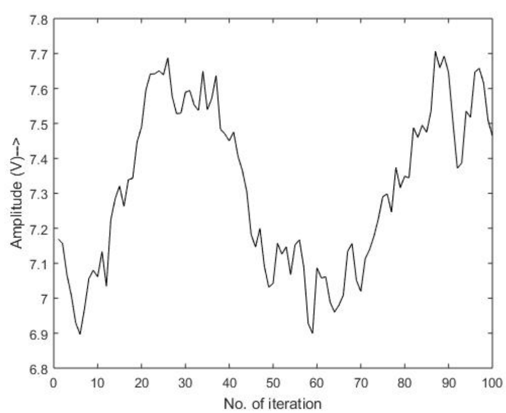
(f)



(g)



(h)



(i)

Figure 2. a) Input sinusoidal signal, b) Estimated output voltage of LPF using EKF, c) Estimated output voltage of LPF using LMS method, d) Estimated output voltage of HPF using EKF, e) Estimated output voltage of HPF using LMS method, f) Estimated output voltage of LPF using EKF with square wave input, g) Estimated output voltage of LPF using LMS method with square wave input, h) Estimated output voltage of HPF using EKF with square wave input. i) Estimated output voltage of HPF using LMS with square wave input.

Table 1. Comparison of SNR value for different method

Input signal	SNR by Proposed method (dB)	SNR by recursive LMS method (dB)
Sinusoidal signal in LPF	43.7696	32.7374
Square wave in LPF	55.994	34.314
Sinusoidal signal in HPF	58.144	26.866
Square wave in HPF	62.650	60.48

5. CONCLUSIONS

The simulations show the output voltage estimation of RC LPF and HPF using EKF. The Table I shows that EKF provide better estimation as compared to LMS method as EKF takes system noise and measurement noise into account. Also, it has the advantage of easy design and implementation.

Acknowledgment

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