On the P -transform of $\overline{H}$ function

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Abstract:

In this paper we study an integral transform ($P-$ transform) introduced by Dilip Kumar [4] ,whose kernel is generalization of pathway model introduced by Mathai [3]. Here we established Theorems related to the P -transform of $\overline{H}$ function whose argument is of the form $x^{\nu-1}\left(c-dx^{\sigma}\right)^{-\mu}$. Next, on account of the most general nature of the function occurring in the integrand of the integral transform, our findings provides interesting unifications and extensions of a number of new and known results . For the sake of illustration, we obtain herein three new special case of the transform involving the functions namely: generalized Wright hypergeometric function Hurwitz -Lerch zeta function, generalized Hypergeometric function.

Keywords: P –transform, $\overline{H}$ –function.

1. Introduction

The paper is devoted to the study of the P-transform or pathway transform given by

$$\left(P_{\nu}^{\rho,\alpha,\beta}f\right)(x)=\int_{0}^{\infty}D_{\rho,\beta}^{\nu,\alpha}(xt)f(t)dt \tag{1}$$

Where $D_{\rho,\beta}^{\nu,\alpha}(x)$ denotes the kernel-function defined as follows

$$D_{\rho,\beta}^{\nu,\alpha}(x)=\int_{0}^{\infty}y^{\nu-1}\left[1+a(\alpha-1)y^{\rho}\right]^{1/[\alpha-1]}e^{-xy^{-\beta}}dy \tag{2}$$

Where $\nu\in C,\beta<0,a>0,\rho\in R,\rho\neq 0,\alpha>1$ In this case, we say that Eq.(1) is a type-2 P transform. Instead of using the kernel function given in Eq. (2), if we use

$$D_{\rho,\beta}^{\nu,\alpha}(x)=\int_{0}^{1/[a(\alpha-1)]^{\frac{1}{\rho}}}y^{\nu-1}\left[1+a(\alpha-1)y^{\rho}\right]^{1/[\alpha-1]}e^{-xy^{-\beta}}dy \tag{3}$$

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We say that Eq.(1) is a type-1 *P* transform.

Further on taking a=1, β=1, transform of Eq. (2) reduces to the following transform

$$P_{\nu}^{\rho,\alpha}\left[f(z);x\right]=\int_{0}^{\infty}D_{\rho}^{\nu,\alpha}(zx)\,f(z)dz, \qquad x>0 \tag{4}$$

where $D_{\rho}^{\nu,\alpha}(z)$ is generalized Krätzel function studied by Kilbas[13,p.835, Eq.(1)].

Again for $\alpha\to 1,$ transform (4) reduces to Krätzel transform [4, p.604,Eq.(5)] defined in the following manner

$$K_{\nu}^{\rho}\left[f(z);x\right]=\int_{0}^{\infty}D_{\rho}^{\nu}(z)f(z)dz\,, \quad x>0 \tag{5}$$

<u>Generalized Krätzel function in terms of H-function</u> [4,p.610,Eqs.(45&46)]:

Let $\rho\in R(\rho\neq 0), \nu\in C,\ \alpha\geq 1$ If $\rho>0,$ Then

$$D_{\rho,\beta}^{\nu,\alpha}(z)=\frac{1}{\rho\left(a(\alpha-1)^{\nu/\rho}\right)\Gamma\left(1/(\alpha-1)\right)}H_{1,2}^{21}\left[\left[a(\alpha-1)\right]^{\beta/\rho}z\left|\begin{matrix}\left(1-\frac{1}{(\alpha-1)}+\frac{\nu}{\rho},\frac{\beta}{\rho}\right)\\ (0,1),\left(\frac{\nu}{\rho},\frac{\beta}{\rho}\right)\end{matrix}\right.\right] \tag{6}$$

If $\rho<0,$ Then

$$D_{\rho,\beta}^{\nu,\alpha}(z)=-\frac{1}{\rho\left(a(\alpha-1)^{\nu/\rho}\right)\Gamma\left(1/(\alpha-1)\right)}H_{1,2}^{21}\left[\left(a(\alpha-1)^{\beta/\rho}z\right)\left|\begin{matrix}\left(1-\frac{\nu}{\rho},-\frac{\beta}{\rho}\right)\\ (0,1),\left(\frac{1}{(\alpha-1)}-\frac{\nu}{\rho},-\frac{\beta}{\rho}\right)\end{matrix}\right.\right] \tag{7}$$

2. THE $\overline{\mathrm{H}}$-FUNCTION

The $\overline{\mathrm{H}}$-function occurring in the present paper was introduced by Inayat Hussain [11] and studied by Buschman and Srivastava [4] and others.. The $\overline{\mathrm{H}}$-function is defined and represented by the following generalized Mellin-Barnes type contour integral [10]:

$$\bar{H}[z]=\bar{H}_{P,Q}^{M,N}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]=\frac{1}{2\pi\omega}\int_L\bar{\phi}(\xi)\,z^{\xi}\,d\xi \tag{8}$$

where $\omega=\sqrt{-1}$

$$\bar{\phi}(\xi)=\frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\left\{\Gamma(1-a_j+\alpha_j\xi)\right\}^{A_j}}{\prod_{j=M+1}^{Q}\left\{\Gamma(1-b_j+\beta_j\xi)\right\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \tag{9}$$

which contains fractional power of some of the gamma functions. The following sufficient conditions for the absolute convergence of the defining integral for $\bar{H}$- function given by Eq.(9) have been recently given by Gupta, Jain and Agrawal [7]:

(i) $|\arg(z)|<1/2\,\Omega\pi$ and $\Omega>0$

(ii) $|\arg(z)|=1/2\,\Omega\pi$ and $\Omega\geq 0$

and (a) $\mu\neq 0$ and the contour L is so chosen that $(c\mu+\lambda+1)<0$

(b) $\mu=0$ and $(\lambda+1)<0$

where

$$\Omega=\sum_{1j=}^{M}\beta_j+\sum_{j=1}^{N}\alpha_jA_j-\sum_{j=M+1}^{Q}\beta_jB_j-\sum_{j=N+1}^{P}\alpha_j \tag{10}$$

$$\mu=\sum_{j=1}^{N}\alpha_jA_j+\sum_{j=N+1}^{P}\alpha_j-\sum_{j=1}^{M}\beta_j-\sum_{j=M+1}^{Q}\beta_jB_j \tag{11}$$

$$\lambda=\mathrm{Re}\left(\sum_{j=1}^{M}b_j+\sum_{j=M+1}^{Q}b_jB_j-\sum_{j=1}^{N}a_jA_j-\sum_{j=N+1}^{P}a_j\right)+\frac{1}{2}\left(-M-\sum_{j=M+1}^{Q}B_j+\sum_{j=1}^{N}A_j+P-N\right) \tag{12}$$

The behavior of the $\bar{H}_{P,Q}^{M,N}[z]$ function for small values of z as recorded by Saxena et.al. [14, p. 112, eq2.3& 2.4] gives

$$\bar{H}_{P,Q}^{M,N}[z]=o[|z|^{\alpha}]\text{ , for small z, where }\alpha=\min_{1\leq j\leq M}\mathrm{Re}\left(b_j/\beta_j\right) \tag{13}$$

Theorem – 1 Let $\nu,\lambda \in C, \beta > 0, \rho \in R, \rho \neq 0, \alpha > 1$, and $\operatorname{Re}(\lambda + \sigma\xi) > 0$ be such that $\operatorname{Re}\left(\frac{\nu+\beta\lambda}{\rho}\right) > 0, \operatorname{Re}\left(\frac{1}{(\alpha-1)} - \frac{(\nu+\beta\lambda)}{\rho}\right) > 0$ when $\rho < 0$, then we have the following result

$$P_{\nu}^{\rho,\alpha,\beta}\left(x^{\lambda-1}\left(c-dx^{\sigma}\right)^{-\mu}\right) = -\frac{\left[a(\alpha-1)\right]^{-(\nu+\beta\lambda)/\rho} c^{-\mu} x^{-\lambda}}{\rho\Gamma(\mu)\Gamma\left[1/(\alpha-1)\right]} H_{3,2}^{2,3}\left[\frac{d}{c}\frac{x^{-\sigma}}{\left[a(\alpha-1)\right]^{\beta\sigma/\rho}}\left|\begin{matrix}(1-\mu,1),(1-\lambda,\sigma),\left(1-\frac{1}{\alpha-1}+\frac{\nu+\beta\lambda}{\rho},-\frac{\beta\sigma}{\rho}\right)\\ (0,1),\left(\frac{\nu+\beta\lambda}{\rho},-\frac{\beta\sigma}{\rho}\right)\end{matrix}\right.\right] \tag{14}$$

Proof:- This theorem can be obtained by observing the following, Let

$$P_{\nu}^{\rho,\alpha,\beta}\left(x^{\lambda-1}\left(c-dx^{\sigma}\right)^{-\mu}\right) = \int_0^{\infty}\int_0^{\infty} y^{\nu-1}\left[1+a(\alpha-1)y^{\rho}\right]^{-1/[\alpha-1]} e^{-xty^{-\beta}} dy\left(t^{\lambda-1}\left(c-dt^{\sigma}\right)^{-\mu}\right) dt \tag{15}$$

Changing the order the order of integration ,left hand side become say

$$\Delta = \int_0^{\infty} y^{\nu-1}\left[1+a(\alpha-1)y^{\rho}\right]^{-1/[\alpha-1]} dy \int_0^{\infty} e^{-xty^{-\beta}} t^{\lambda-1}\left(c-dt^{\sigma}\right)^{-\mu} dt \tag{16}$$

we first express the term $\left(c-dt^{\sigma}\right)^{-\mu}$ occurring on its left hand side in terms of well known Mellin-Barnes contour integral, we have,

$$\Delta = \frac{c^{-\mu}}{2\pi i}\int_L \frac{\Gamma(\mu+\xi)\Gamma(-\xi)}{\Gamma(\mu)}\left(\frac{d}{c}\right)^{\xi}\left(\int_0^{\infty} y^{\nu-1}\left[1+a(\alpha-1)y^{\rho}\right]^{-1/[\alpha-1]} dy \int_0^{\infty} e^{-xty^{-\beta}} t^{\lambda+\sigma\xi-1} dt\right) d\xi \tag{17}$$

Now using the integral representation of gamma function

$$\Delta = \frac{c^{-\mu}}{2\pi i}\int_L \frac{\Gamma(\mu+\xi)\Gamma(-\xi)}{\Gamma(\mu)x^{\lambda+\sigma\xi}}\left(\frac{d}{c}\right)^{\xi}\int_0^{\infty} y^{\nu+\beta\lambda+\beta\sigma\xi-1}\left[1+a(\alpha-1)y^{\rho}\right]^{-1/[\alpha-1]} dy \tag{18}$$

Put $a(\alpha-1)y^{\rho} = u$, we have

$$\Delta=\frac{c^{-\mu}}{2\pi i}\int_L \frac{\Gamma(\mu+\xi)\Gamma(\lambda+\sigma\xi)\Gamma(-\xi)}{\Gamma(\mu)x^{\lambda+\sigma\xi}}\left(\frac{d}{c}\right)^{\xi}\int_0^{\infty}(u)^{\frac{(\nu+\beta\lambda+\beta\sigma\xi)}{\rho}-1}\left[1+u\right]^{-1/[\alpha-1]}du \tag{19}$$

or

$$\Delta=\frac{c^{-\mu}}{\rho\left[a(\alpha-1)\right]^{\nu+\beta\lambda/\rho}\Gamma(\mu)\Gamma 1/[\alpha-1]x^{\lambda}}\frac{1}{2\pi i}\int_L\frac{\Gamma(\mu+\xi)\Gamma(\lambda+\sigma\xi)\Gamma(-\xi)\Gamma\left(\frac{\nu+\beta\lambda+\beta\sigma\xi}{\rho}\right)\Gamma\left(1/[\alpha-1]-\frac{\nu+\beta\lambda+\beta\sigma\xi}{\rho}\right)}{\left[a(\alpha-1)\right]^{\beta\sigma\xi/\rho}}\left(\frac{d}{c}x^{-\sigma}\right)^{\xi}d\xi$$

$$\Delta=\frac{c^{-\mu}x^{-\lambda}}{\rho\left[a(\alpha-1)\right]^{\nu+\beta\lambda/\rho}\Gamma(\mu)\Gamma 1/[\alpha-1]}H_{3,2}^{2,3}\left[\frac{d}{c}\frac{x^{-\sigma}}{\left[a(\alpha-1)\right]^{\beta/\rho}}\left|\begin{matrix}(1-\mu,1),(1-\lambda,\sigma),\left(\frac{\nu+\beta\lambda}{\rho},\frac{\beta\sigma}{\rho}\right)\\(0,1),\left(\frac{1}{\alpha-1}-\frac{\nu+\beta\lambda}{\rho},\frac{\beta\sigma}{\rho}\right)\end{matrix}\right.\right] \tag{20}$$

Theorem−2 Let $\nu,\lambda\in C,\beta<0,z<0,\lambda_1>0,\mu>0,\rho\in R,\rho\neq 0,\alpha>1,\mathrm{Re}(\lambda+\sigma r+\lambda_1\xi)>0$ be such that,

$\mathrm{Re}\left(\frac{\nu+\beta\lambda+\beta\sigma r+\beta\lambda_1\xi}{\rho}\right)>0,\ \mathrm{Re}\left(1/(\alpha-1)-(\nu+\beta\gamma-\beta\delta s)/\rho\right)>0$, when $\rho<0$

$$P_{\nu}^{\rho,\alpha,\beta}\left(x^{\lambda-1}(c-dx^{\sigma})^{-\mu}\overline{H}_{P,Q}^{M,N}\left[zx^{\lambda_1}(c-dx^{\sigma})^{-\mu_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]\right)=$$

$$\sum_{r=0}^{\infty}\frac{c^{-\mu-r}x^{-\lambda-\sigma r}}{\rho\left[a(\alpha-1)\right]^{(\nu+\beta\lambda+\beta\sigma r)/\rho}\Gamma 1/[\alpha-1]}$$

$$\overline{H}_{P+3,Q+1}^{M+1,N+3}\left[c^{-\mu_1}\frac{x^{-\lambda_1}}{\left[a(\alpha-1)\right]^{\beta\lambda_1/\rho}}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(1-\mu+r,\mu_1,1)(1-\lambda+\sigma r,\lambda_1;1),\left(1-\frac{\nu+\beta\lambda+\beta\sigma r}{\rho},\frac{\beta\lambda_1}{\rho};1\right),(a_j,\alpha_j)_{N+1,P}\\\left(\frac{1}{\alpha-1}-\frac{\nu+\beta\lambda+\beta\sigma r}{\rho},\frac{\beta\sigma}{\rho}\right),(b_j,\beta_j)_{1,M}(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right] \tag{21}$$

Proof:-This theorem can be obtained by observing the following,

We first express the $\overline{H}$ function in contour form and term $\left(c-dz^{\sigma}\right)^{-\mu}$ occurring on its left hand side in binomial expansion form,

$$P_{\nu}^{\rho,\alpha,\beta}\left(x^{\lambda-1}\left(c-dx^{\sigma}\right)^{-\mu}\overline{H}_{P,Q}^{M,N}\left[zx^{\lambda_1-1}\left(c-dx^{\sigma}\right)^{-\mu_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]\right)$$

$$=\frac{1}{(2\pi i)}\int_L\bar{\theta}(\xi)\int_0^\infty\int_0^\infty y^{\nu-1}\left[1+a(\alpha-1)y^{\rho}\right]^{-1/[\alpha-1]}e^{-xty^{-\beta}}dy\left(t^{\lambda+\lambda_1\xi-1}\left(c-dt^{\sigma}\right)^{-\mu-\mu_1\xi}\right)dtd\xi \tag{22}$$

Then changing the order the order of integration, we have

$$\Delta=\sum_{r=0}^{\infty}\frac{1}{(2\pi i)}\int_L\bar{\theta}(\xi)\frac{\Gamma(\mu+r+\mu_1\xi)}{r!\,\Gamma(\mu+\mu_1\xi)}\left(\int_0^\infty y^{\nu-1}\left[1+a(\alpha-1)y^{\rho}\right]^{-1/[\alpha-1]}dy\int_0^\infty e^{-xty^{-\beta}}\left(t^{\lambda+\lambda_1\xi+\sigma r-1}\right)dt\right)d\xi \tag{23}$$

using the integral representation of gamma function.

$$\Delta=\sum_{r=0}^{\infty}\frac{c^{-\mu-r}d^{r}}{r!}\frac{1}{(2\pi i)}\int_L\bar{\theta}(\xi)\frac{\Gamma(\lambda+\sigma r+\lambda_1\xi)\Gamma(\mu+r+\mu_1\xi)}{\Gamma(\mu+\mu_1\xi)x^{\lambda+\sigma r+\lambda_1\xi}}c^{-\mu_1\xi}\int_0^\infty y^{\nu+\beta(\lambda+\sigma r+\lambda_1\xi)-1}\left[1+a(\alpha-1)y^{\rho}\right]^{-1/[\alpha-1]}dy \tag{24}$$

Put $a(\alpha-1)y^{\rho}=u$, we have

$$\Delta=\sum_{r=0}^{\infty}\frac{c^{-\mu-r}d^{r}}{r!}\frac{1}{(2\pi i)}\int_L\bar{\theta}(\xi)\frac{\Gamma(\lambda+\sigma r+\lambda_1\xi)\Gamma(\mu+r+\mu_1\xi)c^{-\mu\xi}}{\Gamma(\mu+\mu_1\xi)\left[a(\alpha-1)\right]^{(\nu+\beta\lambda+\sigma r+\beta\lambda_1\xi)/\rho}x^{\lambda+\sigma r+\lambda_1\xi}}\int_0^\infty(u)^{\frac{(\nu+\beta\lambda+\beta\sigma r+\beta\lambda_1\xi)}{\rho}}(1+u)^{-1/[\alpha-1]}du$$

$$\Delta=\sum_{r=0}^{\infty}\frac{c^{-\mu-r}x^{-\lambda-\sigma r}d^{r}}{r!\;\rho\left[a(\alpha-1)\right]^{\nu+\beta\lambda/\rho}\Gamma(\alpha-1)}\frac{1}{(2\pi i)}\int_L\bar{\theta}(\xi)\frac{\Gamma(\lambda+\sigma r+\lambda_1\xi)\Gamma(\mu+r+\mu_1\xi)}{\Gamma(\mu+\mu_1\xi)x^{\lambda_1\xi}}\frac{\Gamma\left(\frac{\nu+\beta\lambda+\beta\sigma r+\beta\lambda_1\xi}{\rho}\right)\Gamma\left(1/[\alpha-1]-\frac{\nu+\beta\lambda+\beta\sigma\lambda\xi}{\rho}\right)}{\left[a(\alpha-1)\right]^{\beta\lambda_1\xi/\rho}}$$

$$\frac{1}{x^{\lambda_1\xi}}c^{-\mu_1\xi}d\xi \tag{25}$$

or

$$\Delta=\frac{c^{-\mu-r}x^{-\lambda-\sigma r}}{\rho\left[a(\alpha-1)\right]^{\nu+\beta\lambda+\beta\sigma r/\rho}\Gamma 1/[\alpha-1]}\bar{H}^{M+1,N+3}_{P+3,Q+1}\left[zc^{-\mu_1}\frac{x^{-\lambda_1}}{\left[a(\alpha-1)\right]^{\beta\lambda_1/\rho}}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{1,N},(1-\mu+r;\mu_1,1)(1-\lambda+\sigma r;\lambda_1,1),\left(1-\frac{\nu+\beta\lambda+\beta\sigma r}{\rho},\frac{\beta\lambda_1}{\rho};1\right)\\ \left(\frac{1}{\alpha-1}-\frac{\nu+\beta\lambda+\beta\sigma r}{\rho},\frac{\beta\sigma}{\rho}\right),(b_j,\beta_j)_{1,M}(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

(26)

Remark-1 The corresponding results for H-function can be obtained by setting $A_1=......=A_N=1=B_{M+1}=........=B_Q$, which is the known result obtained by Dilip kumar [4].

Remark-2 The corresponding results for G-function can be obtained by putting $A_1=......=A_N=1=B_{M+1}=........=B_Q$ and $\alpha_1=...=\alpha_P=1=\beta_1=........=\beta_Q$.

Remark −3$:-A_1=....=A_N=1=B_{M+1}=.....=B_Q$, $M=1, N=P, Q\to Q+1$ and reduce $\bar{H}$ function to generalized Hypergeometric function ${}_PF_Q$ function [14, p.241,Eq.(5.4)], we get a known result obtained by Dilip kumar [4].

3.SPECIAL CASES

(i) If we put $M=1, N=P, Q\to Q+1, Z=-1$, and reduce $\bar{H}$ function to generalized Wright Hypergeometric function ${}_P\bar{\psi}_Q$ [8] function in the Theorem 2 , we easily arrive at the following integral after a little simplification.

$$P_{\nu}^{\rho,\alpha,\beta}\left(x^{\lambda-1}\left(c-dx^{\sigma}\right){}_P\bar{\psi}_Q\left[x^{\lambda_1-1}\left(c-dx^{\sigma}\right)^{-\mu_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,P}\\(b_j,\beta_j;B_j)_{1,Q}\end{matrix}\right.\right]\right)=$$

$$\sum_{r=0}^{\infty}\frac{c^{-\mu-r}x^{-\lambda-\sigma r}}{\rho\left[a(\alpha-1)\right]^{(\nu+\beta\lambda+\beta\sigma r)/\rho}\Gamma 1/[\alpha-1]}\bar{H}^{2,P+3}_{P+3,Q+2}\left[-c^{-\mu_1}\frac{x^{-\lambda_1}}{\left[a(\alpha-1)\right]^{\beta\lambda_1/\rho}}\left|\begin{matrix}(1-a_j,\alpha_j;A_j)_{1,P},(1-\mu+r,\mu_1;1)(1-\lambda+\sigma r,\lambda_1;1),\left(1-\frac{\nu+\beta\lambda+\beta\sigma r}{\rho},\frac{\beta\lambda_1}{\rho};1\right)\\(0,1),\left(\frac{1}{\alpha-1}-\frac{\nu+\beta\lambda+\beta\sigma r}{\rho},\frac{\beta\sigma}{\rho}\right),(1-b_j,\beta_j;B_j)_{1,Q}\end{matrix}\right.\right]$$

(27)

(ii) If we put $M=1, N=P, Q\to Q+1, Z=-1$, and reduce $\overline{H}$ function to generalized Hurwitz lerch zeta function $\varphi_{\eta,\gamma,\psi}$ function[12] in the Theorem 2 , we easily arrive at the following integral after a little simplification.

$$\left(P_{\nu}^{\rho,\alpha,\beta} x^{\lambda-1}\left(c-dx^{\sigma}\right)\varphi_{\eta,\gamma,\psi}\left[x^{\lambda_1-1}\left(c-dx^{\sigma}\right)^{-\mu_1}\right]\right)(x)=\frac{\Gamma(\delta)\Gamma(\gamma)}{\Gamma(\psi)}\frac{c^{-\mu-r}x^{-\lambda-\sigma r}}{\rho\left[a(\alpha-1)\right]^{(\nu+\beta\lambda+\beta\sigma r)/\rho}\Gamma 1/[\alpha-1]}$$

$$\overline{H}_{6,5}^{2,6}\left[-c^{-\mu_1}\frac{1}{x^{\lambda_1}\left[a(\alpha-1)\right]^{\beta\lambda_1/\rho}}\middle|\begin{matrix}(1-\eta,1;p),(1-\delta,1;1),(1-\gamma,1;1),(1-\mu+r,\mu_1,1)(1-\lambda+\sigma r,\lambda_1;1),\left(1-\frac{\nu+\beta\lambda+\beta\sigma r}{\rho},\frac{\beta\lambda_1}{\rho};1\right)\\(0,1),(1-\psi,1;1),(\eta,1;p),\left(\frac{1}{\alpha-1}-\frac{\nu+\beta\lambda+\beta\sigma r}{\rho},\frac{\beta\sigma}{\rho}\right),(b_j,\beta_j;B_j)_{1,Q}\end{matrix}\right]$$

(28)

(iii) If we put $M=1, N=P, Q\to Q+1$, and reduce $\overline{H}$ function to generalized Hyper-geometric function [14, p.241,Eq.(5.4)], in the Theorem 2 , we easily arrive at the following integral after a little simplification.

$$\left(P_{\nu}^{\rho,\alpha,\beta} x^{\lambda-1}\left(c-dx^{\sigma}\right)^{-\mu}{}_P\overline{F}_Q\left[x^{\lambda_1-1}\left(c-dx^{\sigma}\right)^{-\mu_1}\right]\right)(x)=$$

$$\frac{\prod_{j=1}^{Q}\left[\Gamma(b_j)\right]^{B_j}}{\prod_{j=1}^{P}\left[\Gamma(a_j)\right]^{A_j}}\frac{c^{-\mu-r}x^{-\lambda-\sigma r}}{\rho\left[a(\alpha-1)\right]^{(\nu+\beta\lambda+\beta\sigma r)/\rho}\Gamma 1/[\alpha-1]2\pi i}$$

$$\overline{H}_{P+3,Q+2}^{2,P+3}\left[-c^{-\mu_1}\frac{x^{-\lambda_1}}{\left[a(\alpha-1)\right]^{\beta\lambda_1/\rho}}\middle|\begin{matrix}(1-a_j,1;A_j)_{1,P}(1-\mu+r,\mu_1,1)(1-\lambda+\sigma r,\lambda_1;1),\left(1-\frac{\nu+\beta\lambda+\beta\sigma r}{\rho},\frac{\beta\lambda_1}{\rho};1\right)\\(0,1),\left(\frac{1}{\alpha-1}-\frac{\nu+\beta\lambda+\beta\sigma r}{\rho},\frac{\beta\sigma}{\rho}\right),(1-b_j,1;B_j)_{1,Q}\end{matrix}\right]$$

(29)

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