# A STUDY OF HIV MODEL USING NADM METHOD

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**Abstract:** This paper presents a process to obtain the solution of dynamic model for HIV infection of $CD4^+$ T cells. For analytical solution of non linear differential equation, we are using Natural adomian decomposition method (NADM). This method gives reliable and effective solution of HIV model. In this research paper, we are using only a few iterations to find exact solution by NADM.

**Keywords:** Natural Transform, Natural Adomian Decomposition Method (NADM), HIV infection, CD4+ T cells.

## 1. Introduction

HIV (human immune deficiency virus) is an uncommon type of retrovirus called lent virus that causes acquired immunodeficiency syndrome[1] (AIDS).HIV destroys the $CD4^+$T cells lymphocytes of the immune system, which help the immune system fight off infections. In the cell, HIV produces virus particles by converting viral RNA into DNA and then making many RNA. Without treatments, HIV advances in stages and three stages of HIV infections are: Acute HIV infection, Clinical latency and AIDS [2-3] (acquired immune deficiency syndrome).

In this paper, we are solving the HIV classic model by Natural adomian decomposition method (NADM) and explain the solution in series .The Current status of HIV infection [4] in the world is given in the table-1.

| | | |
|---|---|---|
| No. of people living with HIV in 2016 | Total- 36.7 million<br>Adults – 34.5 million<br>Women- 17.8 million<br>Men – 16.7 million<br>Children (<15 years)- 2.1 million | {Avg 30.8 -42.9 million}<br>{Avg 28.8 - 40.2 million}<br>{Avg 15.4 – 20.3 million}<br>{Avg 14.0 – 19.5 million}<br>{Avg 1.7 -2.6 million} |
| People newly infected with HIV in 2016 | Total – 1.8 million<br>Adults- 1.7 million<br>Children(<15 years) -160,000 | {Avg 1.6 – 2.1million }<br>{Avg 1.4-1.9 million}<br>{Avg 100,000 -220,000} |
| Aids deaths in 2016 | Total- 1.0 million<br>Adults – 890,000<br>Children(<15 years)- 120,000 | {Avg 830000-1.2 million}<br>{Avg 740,000-1.1million}<br>{Avg 79,000-160,000 million} |

(Table- 1 Current status of HIV in the world)

In 2016, 500 people were still getting infected with HIV every day.

**HIV classic model:** Our model is governed by the following ordinary differential equation:

$$\left.\begin{aligned} \frac{dH}{dt} &= a - bH + qH\left(1 - \frac{H+I}{H_{max}}\right) - K_1 VH \\ \frac{dI}{dt} &= K_2 VH - cI \\ \frac{dV}{dt} &= McI - dV - K_1 VH \end{aligned}\right\} \qquad (*)$$

with initial conditions $H(0) = q_{1,0}$ ; $I(0) = q_{2,0}$ ; $V(0) = q_{3,0}$ .

Here,
H =Healthy cells
I = Infected cells
V = Virus

**Parameters specification:**
All the parameters and their values used for model throughout this paper [5]. So we set,

(Table-2 Description of parameters and their values)

| Parameter and variables | Description | Values |
|---|---|---|
| A | Source term for uninfected $CD4^+$ T cells | 0.1 per day |
| B | Natural death rate of $CD4^+$T cells | 0.02 per day |
| Q | Growth rate of $CD4^+$ T cells population | 0.3 per day |
| $H_{max}$ | Maximal population level of $CD4^+$ T-cells | 1500 |
| $K_1$ | Infection rate | 0.0027 |
| $K_2$ | Rate of infected cells become active | 0.002 |
| C | Blanket death rate of $CD4^+$ T cells | 0.3 per day |
| M | No. Of virus produced by infected $CD4^+$ T cells | 0 |
| D | Death rate of free virus | 2.4 |

To solve the model for HIV infection of $CD4^+$ T cells, we are concerned to expand the application of the analytic NADM.

**Basic Idea of The Integral Transform Method**: Assume we have a function $f(t)$, $t \in (-\infty, \infty)$, and then the general integral transform is defined as follows:

$$\mathcal{I}[f(t)](s) = \oint_{-\infty}^{\infty} k(s,t).f(t)dt \tag{1}$$

here, is the kernel of the transform. Where $k(s,t)$ represent the kernel of the transform, $s$ is the real (complex) number, which is independent of $t$ .

**Definitions of the Natural Transform:** The Natural transform [12] of the function $f(t) > 0$ and $f(t) = 0$ for $t < 0$ is defined by

$$N^+[f(t)] = R(s,u) = \int_0^{\infty} e^{-st} f(ut)dt; \qquad s > 0, \;\; u > 0. \tag{2}$$

Where $N^+[f(t)]$ is the natural transformation of the time function $f(t)$ and the variables $s$ and $u$ are the natural transform variables, provided the function $f(t)$ is defined in the set $A$ by

$$\mathrm{A} = \left\{ f(t) \colon \exists\, \mathrm{M}, k_1, k_2 > 0\,, |\, f(t)| < \mathrm{M} e^{\frac{|t|}{k_j}}\,, \colon if\ \mathrm{t} \in (-1)^j \times [0,\infty) \right\}, \tag{3}$$

where $M$ is a constant of finite number, $k_1$ and $k_2$ may be finite or infinite.

Also the inverse of Natural transform is defined by

$$N^{-1}[R(s,u)] = f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{st/u} R(s,u)ds \tag{4}$$

**Properties Natural transform:**

For the function $f(t)\epsilon A$; multiplied with shift function $t^n$, Natural transform of $f(t)$is given by

$$N^{+}[t^n f(t)] = (-u)^n \frac{d^n}{ds^n} R(s,u) \tag{5}$$

If $f^n(t)$ is $n^{th}$ derivative of function $f(t)$ w.r.t. $t^n$, then Natural transform is defined by

$$N^{+}[t^n f(t)] = (u)^n \frac{d^n}{ds^n} R(s,u) \tag{6}$$

**The Algorithm of the Adomian Decomposition Method:**

To solve ordinary and partial non linear differential equation, we use ADM which is semi – analytical method. When we decompose a nonlinear portion a series is obtained, that is known as Adomian polynomials. By the use of Adomian polynomial [9], with the help of recursive relationship, a solution is obtained in the form of series.

The algorithm of the method is given by

**Step 1**. Let have a non linear equation,

$W = w\,;$ (7)

$w$ may be the function and value. Where, W is a non linear operator.

**Step 2**. The equation in operator form is given as $N\,y(t) + M\,y(t) + P\,y\,(t) = w\,(t)$ (8)

**Step 3**. Applying $N^{-1}$ inverse operator on both side

$N^{-1}N\,y(t) + N^{-1}M\,y(t) + N^{-1}P\,y(t) = w$ (9)

Eq.(9) becomes, $y(t) = w(t) - N^{-1}M\,y(t) - N^{-1}P\,y(t)$ (10)

**Step 4**. Let the unknown function $y(t)$ given as infinite series,

$y(t) = \sum_{n=0}^{\infty} y_n\,(t).$ (11)

**Step5.** Let $y_0 = w(t)$ (12)

To obtain other terms, use recursive relationship. Here, non- linear term $Py(t)$ can be written as

$P\,y(t) = \sum_{n=0}^{\infty} Q_n$ (13)

**Step 6**. To get the value of $Q_n$, we put a grouping parameter $\tau$. Thus we have

$Q_n = \frac{1}{n!}\frac{d^n}{d\tau^n}\,P\,y(\tau)$ and $\tau = 0\,.$ (14)

Hence, ADM gives a convergent series solution [9-10]. This solution is absolutely and uniformly convergent.

**2. Analytical solution of HIV infection of CD4 $^+$T cell:**

According to Eq.(*) we have,

$$\frac{dH}{dt} = a - bH + q\,H\left(1 - \frac{H+I}{H_{max}}\right) - K_1 VH$$

$$\frac{dI}{dt} = K_2 VH - cI$$

$$\frac{dV}{dt} = McI - dV - K_1 VH\,.$$

Apply Natural transform on both sides in the above equations, we obtain

$$N\left\{\frac{dH}{dt}\right\} = N\left\{a - bH + q\,H\,\left(1 - \frac{H+I}{H_{max}}\right) - K_1VH\right\}$$

$$N\left\{\frac{dH}{dt}\right\} = \left\{N(a) - N(bH) + N\{q\,H\,\left(1 - \frac{H+I}{H_{max}}\right)\} - N(K_1VH)\right\}$$

$$N\left\{\frac{dH}{dt}\right\} = N\{a\} - N\{bH\} + N\{qH\} - N\left\{\frac{qH^2}{H_{max}}\right\} - N\left\{\frac{qHI}{H_{max}}\right\} - N\{K_1VH\}. \quad (15)$$

$$N\left\{\frac{dI}{dt}\right\} = N\{K_2VH - cI\}.$$

$$N\left\{\frac{dV}{dt}\right\} = N\{McI - dV - K_1VH\}$$

$$N\left\{\frac{dI}{dt}\right\} = N\{K_2VH\} - N\{cI\} \quad (16)$$

$$N\left\{\frac{dV}{dt}\right\} = N\{McI\} - N\{dV\} - N\{K_1VH\}. \quad (17)$$

From Eq.(15), we get

$$\frac{s}{u}\{H(s,u)\} - \frac{1}{u}H(0) \quad = aN\{1\} - bN\{H\} + qN\{H\} - \frac{q}{H_{max}}N\{H^2\} - \frac{q}{H_{max}}N\{HI\} - K_1N\{VH\}$$

$$\frac{s}{u}\{H(s,u)\} = \frac{1}{u}q_{1,0} + \; a.\frac{1}{s} - bN\{H\} + qN\{H\} - \frac{q}{H_{max}}N\{H^2\} - \frac{q}{H_{max}}N\{HI\} - K_1N\{VH\}$$

$$H(s,u) = \frac{1}{s}q_{1,0} + \; a.\frac{u}{s^2} - \frac{u}{s}bN\{H\} + \frac{u}{s}qN\{H\} - \frac{u}{s}\frac{q}{H_{max}}N\{H^2\} - \frac{u}{s}\frac{q}{H_{max}}N\{HI\} - \frac{u}{s}K_1N\{VH\}$$

$$H(s,u) = \frac{1}{s}q_1 + \; a.\frac{u}{s^2} - \frac{u}{s}bN\{H\} + \frac{u}{s}qN\{H\} - \frac{u}{s}\frac{q}{H_{max}}N\{H^2\} - \frac{u}{s}\frac{q}{H_{max}}N\{HI\} - \frac{u}{s}K_1N\{VH\}. \quad (18)$$

Again from Eq.(16),

$$N\left\{\frac{dI}{dt}\right\} = N\{k_2VH\} - N\{cI\}$$

using Natural transform properties ,we get

$$\frac{s}{u}\{I(s,u)\} - \frac{1}{u}I(0) = K_2N(VH) - cN$$

$$\frac{s}{u}\{I(s,u)\} = \frac{1}{u}q_{2,0} + K_2N(VH) - cN(I)$$

$$\{I(s,u)\} = \frac{1}{s}q_{2,0} + \frac{u}{s}K_2N(VH) - \frac{u}{s}cN(I)$$

$$\{I(s,u)\} = \frac{1}{s}q_2 + \frac{u}{s}K_2N(VH) - \frac{u}{s} \quad . \quad (19)$$

Apply Natural transform properties on Eq.(17)

$$\frac{s}{u}\{V(s,u)\} - \frac{1}{u}V(0) = McN(I) - K_1N(VH) - dN(V)$$

$$\frac{s}{u}\{V(s,u)\} = \frac{1}{u}q_{3,0} + McN(I) - K_1N(VH) - dN(V)$$

$$\{V(s,u)\} = \frac{1}{s}q_{3,0} + \frac{u}{s}McN(I) - \frac{u}{s}K_1N(VH) - \frac{u}{s}dN(V)$$

$$\{V(s,u)\} = \frac{1}{s}q_3 + \frac{u}{s}McN(I) - \frac{u}{s}K_1N(VH) - \frac{u}{s}dN(V) \quad (20)$$

Now, we assume that

$$H = \sum_{n=0}^{\infty} H_n \quad I = \sum_{0}^{\infty} I_n , V = \sum_{0}^{\infty} V_n \ , \quad E = H^2, \quad F = HI, G = VH , E = \sum_{0}^{\infty} E_n \quad , F = \sum_{0}^{\infty} F_n \ .$$

We put the assumed values in Eq.(18),we arrive

$$\{N \textstyle\sum_{n=0}^{\infty} H_n\} = \frac{q_1}{s} + a\frac{u}{s^2} - \frac{u}{s} bN\{\sum_0^{\infty} H_n\} + \frac{u}{s} qN\{\sum_0^{\infty} H_n\} - \frac{u}{s}\frac{q}{H_{max}} N\{\sum_0^{\infty} E_n\} - \frac{u}{s}\frac{q}{H_{max}} N\{\sum_0^{\infty} F_n\} - \frac{u}{s} K_1 N\{\sum_0^{\infty} G_n\} . \quad (21)$$

Again ,we put the assumed values in Eq.(19) ,we have

$$\{N \textstyle\sum_{n=0}^{\infty} I_n\} = \frac{q_2}{s} + \frac{u}{s} K_2 N\{\sum_0^{\infty} G_n\} - \frac{u}{s} cN\{\sum_0^{\infty} I_n\} \quad (22)$$

We put the assumed values in Eq.(20), we obtain

$$\{V \textstyle\sum_{n=0}^{\infty} V_n\} = \frac{q_3}{s} + \frac{u}{s} McN\{\sum_0^{\infty} I_n\} - \frac{u}{s} K_1 N\{\sum_0^{\infty} G_n\} - \frac{u}{s} dN\{\sum_0^{\infty} V_n\} . \quad (23)$$

Here $E_n$ , $F_n$ , $G_n$ are Adomian polynomials.

These Adomian polynomials are given by following manner,

$$E_n = \begin{array}{c} E_{0=}{H_0}^2 \\ E_{1=}2H_0H_1 \\ E_{2=}2H_0H_2 + {H_1}^2 \\ E_{3=}2H_0H_3 + 2H_1H_2 \\ E_{4=}2H_0H_4 + 2H_1H_3 + {H_2}^2 \\ E_{5=}2H_0H_5 + 2H_1H_4 + 2H_2H_3 \ . \end{array}$$

$$F_n = \begin{array}{c} F_{0=}H_0I_1 \\ F_{1=}H_1I_0 + H_0I_1 \\ F_{2=}H_2I_0 + H_1I_1 + H_0I_2 \\ F_{3=}H_3I_0 + H_2I_1 + H_1I_2 + H_0I_3 \\ F_{4=}H_4I_0 + H_3I_1 + H_2I_2 + H_1I_3 + H_0I_4 \\ F_{5=}H_5I_0 + H_4I_1 + H_3I_2 + H_2I_3 + H_1I_4 + H_0I_5 \ . \end{array}$$

$$G_n = \begin{array}{c} G_{0=}V_0H_0 \\ G_{1=}V_1H_0 + V_0H_1 \\ G_{2=}V_2H_0 + V_1H_1 + V_0H_2 \\ G_{3=}V_3H_0 + V_2I_1 + V_1H_2 + V_0H_3 \\ G_{4=}V_4H_0 + V_3H_1 + V_2H_2 + V_1H_3 + V_0H_4 \\ G_{5=}V_5H_0 + V_4H_1 + V_3H_2 + V_2H_3 + V_1H_4 + V_0H_5. \end{array}$$

Using the above values in Eq.(21),Eq.(22) and Eq.(23) respectively and we have following solutions[9-10].
From Eq.(21),we get

$$N\{H_0\} = \frac{q_{1,0}}{s} + \frac{u}{s^2}a$$

$$N\{H_1\} = \frac{-u}{s} bN\{H_0\} + \frac{u}{s} qN\{H_0\} - \frac{u}{s}\frac{q}{H_{max}} N\{E_0\} - \frac{u}{s}\frac{q}{H_{max}} N\{F_0\} - \frac{K_I u}{s} N\{G_0\}$$

$$N\{H_2\} = \frac{-u}{s} bN\{H_1\} + \frac{u}{s} qN\{H_1\} - \frac{u}{s}\frac{q}{H_{max}} N\{E_1\} - \frac{u}{s}\frac{q}{H_{max}} N\{F_n\} - \frac{K_I u}{s} N\{G_n\}$$

$$N\{H_{n+1}\} = \frac{-u}{s} bN\{H_n\} + \frac{u}{s} qN\{H_n\} - \frac{u}{s}\frac{q}{H_{max}} N\{E_n\} - \frac{u}{s}\frac{q}{H_{max}} N\{F_n\} - \frac{K_I u}{s} N\{G_n\}. \quad (24)$$

From Eq.(22),we obtain

$$N\{I_0\} = \frac{q_{2,0}}{s}$$

$$N\{I_1\} = \frac{u}{s}K_2N\{G_0\} - \frac{u}{s}cN\{I_0\}$$

$$N\{I_2\} = \frac{u}{s}K_2N\{G_1\} - \frac{u}{s}cN\{I_1\}$$

$$N\{I_{n+1}\} = \frac{u}{s}K_2N\{G_n\} - \frac{u}{s}cN\{I_n\} \quad . \tag{25}$$

From Eq.(23),we get $N\{V_0\} = \frac{q_{3,0}}{s}$

$$N\{V_1\} = \frac{Mcu}{s}N\{I_0\} - \frac{u}{s}K_1N\{G_0\} - \frac{u}{s}dN\{V_0\}$$

$$N\{V_{n+1}\} = \frac{Mcu}{-s}N\{I_n\} - \frac{u}{s}K_1N\{G_n\} - \frac{u}{s}dN\{V_n\} \tag{26}$$

**Result:**

Let us take

$$\text{H}(0) = q_{1,0} = 0.01, \quad I(0) = q_{2,0} = 0.01, \quad V(0) = q_{3,0} = 0.01\,.$$

And we use Natural inverse transform on both the side of the above equations, we get following values of

$$\text{H(t)} = 0.01 + 0 + 3979529\text{t} + 0.148935343t^2 + 0.000075900221t^3 + 0.0369955846t^4 + 0.007398668t^5 - 0.0000001655557t^6$$

$$\text{I (t)} = 0.01 + 0.000019\text{t} + 0.0000672t^2 - 0.00004767784t^3 - 0.00001789813t^4 - 0.0000000085222668t^5 + 0.000000000176t^6$$

$$\text{V(t)} = 0.01 - 0.2407253\text{t} + 0.28792736t^2 + 0.00019447152t^3 + 0.000000000443t^4$$

## Discussion and Conclusion

We have applied NADM for a classical model of HIV CD4$^+$ T cells in this paper. This paper explains a mathematical solution in the polynomial form. Our aim has been to give suggestion that without linearization, the mathematical solution of non linear differential equation systems can be easily obtained. The illustration proves that the NADM is a suitable method to solve non- linear systems easily.

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