Analysis of Flow of a MHD Casson Fluid over an exponentially stretching sheet

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Abstract: *The present paper contains the boundary layer flow and heat transfer of a non- Newtonian MHD Casson fluid over an exponentially stretching permeable surface. Using similarity transformations the governing partial differential equations are converted into ordinary differential equations, and numerical solutions to these equations are obtained using Keller Box method. It is observed that the effect of Casson parameter is to decrease velocity.*

Keywords: Casson fluid, MHD, heat transfer , Keller Box method, exponentially stretching sheet.

1. Introduction

Sakiadis[1] examined the boundary layer flow over a continuous solid surface. L.J.Crane[2] studied flow past a stretching surface. Magyari et. al.[3] studied the heat and mass transfer on boundary layer flow due to an exponentially stretching sheet. Elbashbeshy[4] investigated the flow of an exponentially stretching sheet. Sajid et al[5] examined the influence of thermal radiation on boundary layer flow due to exponentially stretching sheet using HAM. Bidin et al[6] analysed the effect of thermal radiation on the study of laminar two-dimensional boundary layer flow and heat transfer over an exponentially stretching sheet. Hymavathi et al[7] studied numerically the flow and heat transfer of a Casson Fluid over an exponentially permeable Stretching Surface using Keller Box method . Swathy Mukhopadhyay[8] discussed the Casson fluid flow and heat transfer at an exponentially stretching permeable surface using Shooting method.

Swathi Mukhopadhyay[9] studied MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium. Swathi Mukhopadhyay[10] also examined the slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation.

In this present study of work, numerical solution to MHD Casson fluid flow over an exponentially stretching surface is examined. The Keller Box method is discussed for numerical computation. The velocity profiles were examined.

2 Equations of Motion

Consider the boundary layer flow of a steady incompressible viscous fluid past a flat sheet coinciding with the plane y = 0. The fluid flow is confined to y > 0. Two equal and opposite forces are applied along the *x* axis so that the wall is stretched keeping the origin fixed. The rheological equation of state for an isotropic and incompressible Casson fluid is

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + p_y/\sqrt{2\pi}\right)e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + p_y/\sqrt{2\pi_c}\right)e_{ij}, & \pi < \pi_c \end{cases} \tag{1}$$

Here $\pi = e_{ij}e_{ij}$ and e_{ij}, is the (i,j)th component of the deformation rate, π is the product of the component of the deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluid, and P_y is the yield stress of the fluid. The continuity, momentum and energy equations governing such a type of flow problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 u}{\rho} \tag{3}$$

Where, u and v are the components of velocity respectively in the *x* and *y* directions, v is the kinematic viscosity, ρ is the fluid density (assumed constant), $\beta = \mu_B\sqrt{2\pi_c}/p_y$ is the non-Newtonian parameter of the Casson fluid, *k* is the thermal diffusion coefficient of the fluid.

Boundary Conditions.

The appropriate boundary conditions for the problem are given by

$$u = U,\ v = -V_0(x) \text{ at } y = 0 \tag{4}$$

$$\text{as } y \to \infty,\ u \to 0 \tag{5}$$

Here $U = U_0 e^{x/L}$ is the stretching velocity, U_0 is the reference velocity and temperature respectively. $V(x) = V_0 e^{x/2L}$ is a special type of velocity at the wall with V_0 as constant. The meaning of V_0 :V(x)>0 is the velocity suction and V(x)<0 is the velocity blowing.

Equations (2)-(3) can be made dimensionless by introducing the following change of variables

$$□ = \sqrt{\frac{□_0}{2□□}}\, □^{\frac{□}{2□}}\, □ \quad \text{(6a)}$$

$$□ = □_0 □^{\frac{□}{□}} □'(□) \quad \text{(6b)}$$

$$□ = -\sqrt{\frac{□□_0}{2□}}\, □^{\frac{□}{2□}} [□\ □ + □\ ']\quad \text{(6c)}$$

The dimensionless problem satisfies

$$\left(1 + \frac{1}{□}\right) □''' + □□'' - 2□'^2 - □□' = 0 \quad \text{(7)}$$

and the boundary conditions take the following form

$$□□\ □ = 0,\ □ = □,\ □' = 1 \quad \text{(8)}$$

$$□□\ □ \to \infty,\ □' \to 0 \quad \text{(9)}$$

Where $□ = □ / \sqrt{\frac{□_0 □}{2□}}_0$ >0 or (<0) is the suction or (blowing) parameter and $□□ = \frac{□}{□}$ is the Prandtl number.

Numerical Procedure

Equation subject to boundary conditions is solved numerically using an implicit-finite difference scheme known as Keller Box method, as described by Cebeci and Bradshaw[11]. The steps followed are

1. Reduce (7) to a first order equation
2. Write the difference equations using central differences
3. Linearize the resulting algebraic equation by Newton's method and write in matrix vector form
4. Use the block tridiagonal elimination technique to solve the linear system.

Consider the flow equation and concentration equations(7)
and the boundary conditions (8) and (9)

Introduce $f' = p$, (10)

$p' = q$, (11)

eqn(7) reduces to

$$\left(1 + \frac{1}{□}\right) □' + □□ - 2□^2 - □□ = 0 \quad \text{(12)}$$

consider the segment η_{j-1}, η_j with $\eta_{j-1/2}$ as the mid point $\eta_0 = 0$, $\eta_j = \eta_{j-1} + h_j$, $\eta_j = \eta_\infty$ (13)

where h_j is the Δη spaces and j=1,2,……..J is a sequence number that indicates the coordinate locations.

Introducing finite differences

$$\frac{f_j - f_{j-1}}{h_j} = \frac{p_j + p_{j-1}}{2} = p_{j-1/2} \tag{14}$$

$$\frac{p_j - p_{j-1}}{h_j} = \frac{q_j + q_{j-1}}{2} = q_{j-1/2} \tag{15}$$

$$\left(1+\frac{1}{\beta}\right)\frac{q_j - q_{j-1}}{h_j} + \left(\frac{f_j+f_{j-1}}{2}\right)\left(\frac{q_j+q_{j-1}}{2}\right) - 2\left(\frac{p_j+p_{j-1}}{2}\right)^2 - M\left(\left(\frac{p_j+p_{j-1}}{2}\right)\right) = 0 \tag{16}$$

Newton's method

Linearizing the non linear system of equations (14) to (16)

Introduce

$$f_j^{(k+1)} = f_j^{(k)} + \delta f_j^{(k)}$$

$$p_j^{(k+1)} = p_j^{(k)} + \delta p_j^{(k)}$$

$$q_j^{(k+1)} = q_j^{(k)} + \delta q_j^{(k)}$$

Substitute in equations (14) to (16)

Write
$$\delta f_j - \delta f_{j-1} \square \frac{h_j}{2}(\delta p_j + \delta p_{j-1}) = (r_1)_{j-\frac{1}{2}} \tag{17}$$

$$\delta p_j - \delta p_{j-1} \square \frac{h_j}{2}(\delta q_j + \delta q_{j-1}) = (r_2)_{j-\frac{1}{2}} \tag{18}$$

$$(a_1)_j \delta q_j + (a_2)_j \delta q_{j-1} + (a_3)_j \delta f_j + (a_4)_j \delta f_{j-1} + (a_5)_j \delta p_j + (a_6)_j \delta p_{j-1} = (r_4)_{j-\frac{1}{2}} \tag{19}$$

where,

$$\begin{aligned}
(a_1)_j &= 1 + \frac{\beta h_j}{4(\beta+1)}(f_j + f_{j-1}) \\
(a_2)_j &= (a_1)_j - 2.0 \\
(a_3)_j &= \frac{\beta h_j}{4(\beta+1)}(q_j + q_{j-1}) \\
(a_4)_j &= (a_3)_j \\
(a_5)_j &= -\frac{\beta h_j}{(\beta+1)}((p_j + p_{j-1}) + M) \\
(a_6)_j &= (a_5)_j
\end{aligned} \tag{20}$$

and

$$(r_1)_j = f_{j-1} - f_j + \frac{h_j}{2}\left(p_j + p_{j-1}\right)$$
$$(r_2)_j = p_{j-1} - p_j + \frac{h_j}{2}\left(q_j + q_{j-1}\right)$$
$$(r_3)_j = g_{j-1} - g_j + \frac{h_j}{2}\left(t_j + t_{j-1}\right) \qquad (21)$$
$$(r_4)_j = q_{j-1} - q_j - \frac{\beta h_j}{4(\beta+1)}\left(f_j + f_{j-1}\right)\left(q_j + q_{j-1}\right) + \frac{\beta h_j}{2(\beta+1)}\left(p_j + p_{j-1}\right)^2 + M\frac{\beta h_j}{2(\beta+1)}\left(p_j + p_{j-1}\right)$$
$$(r_5)_j = t_{j-1} - t_j - \frac{\Pr h_j}{4}\left(f_j + f_{j-1}\right)\left(t_j + t_{j-1}\right) + \frac{\Pr h_j}{4}\left(p_j + p_{j-1}\right)\left(g_j + g_{j-1}\right)$$

Taking j=1,2,3…

The system of equations becomes

$$[A_1][\delta_1]+[C_1][\delta_2]=[r_1]$$
$$[B_2][\delta_1]+[A_2][\delta_2]+[C_2][\delta_3] = [r_2] \qquad (22)$$
$$\ldots..[B_{J-1}][\delta_1]+[A_{J-1}][\delta_2]+[C_{J-1}][\delta_3] = [r_{J-1}]$$
$$[B_J][\delta_{J-1}]+[A_J][\delta_J]= [r_J]$$

Where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ d & 0 & d \\ (a_2)_1 & (a_3)_1 & (a_1)_1 \end{bmatrix} \quad A_j = \begin{bmatrix} d & 1 & 0 \\ 1 & 0 & 0 \\ (a_1)_j & (a_3)_j & (a_6)_j \end{bmatrix}$$

$$B_j = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & d \\ 0 & (a_4)_j & (a_2)_j \end{bmatrix} \qquad C_j = \begin{bmatrix} d & 0 & 0 \\ 1 & 0 & 0 \\ (a_5)_1 & 0 & 0 \end{bmatrix} \qquad (23)$$

The Block Elimination Method

The linearized differential equations of the system has a block diagonal structure. This can be written in tri diagonal matrix form as

$$\begin{bmatrix} [A_1] & [C_1] & & & \\ [B_2] & [A_2] & [C_2] & & \\ & & \vdots & & \\ & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\ & & & [B_J] & [A_J] \end{bmatrix} \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{J-1}] \\ [\delta_J] \end{bmatrix} = \begin{bmatrix} [r_1] \\ [r_2] \\ \vdots \\ [r_{J-1}] \\ [r_J] \end{bmatrix} \qquad (24)$$

This is of the form A δ = r (25)

To solve the above system

Write [A] = [L] [U] (26)

where

$$L = \begin{bmatrix} [\alpha_1] & & & \\ [\beta_2] & [\alpha_2] & & \\ & & \ddots & \\ & & [\alpha_{j-1}] & \\ & & [\beta_J] & [\alpha_J] \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} [I] & [\Gamma_1] & & \\ & [I_2] & [\Gamma_2] & \\ & & \ddots & \\ & & [I] & [\Gamma_{J-1}] \\ & & & [I] \end{bmatrix} \qquad (27)$$

Where [I] is the identity matrix

$[\alpha_i][\Gamma_i]$ are determined by the following equations

$[\alpha_1] = [A_1]$

$[A_1][\Gamma_1] = [C_1]$

$[\alpha_j] = [A_j]-[B_j][\Gamma_{j-1}]$ j=2,3,. ....,J

$[\alpha_j][\Gamma_j] = [C_j]$ j=2,3,..........,J-1

Substituting (26) in (25)

LUδ = r

Let U δ = W

then LW = r

$$\text{where } W = \begin{bmatrix} [w_1] \\ [w_2] \\ \vdots \\ [w_{j-1}] \\ [w_J] \end{bmatrix}$$

Now $[\alpha_1]\,[w_1] = [r_1]$

$[\alpha_j]\,[w_J] = [r_J]-[B_J][W_{j-1}]$ for $2 \le j \le J$

Once the elements of W are found, substitute in Lδ=W and solve for δ

$[\delta_J] = [W_J]$

$[\delta_J] = [W_J]-[\Gamma_J][\delta_{J+1}]$, $1 \leq j \leq J-1$

These calculations are repeated until some convergence criterion is satisfied and we stop the calculations when $|\delta g_0^{(i)}| \leq \varepsilon$, where ε is very small prescribed value taken to be ε = 0.0001.

3. Results and discussion

The velocity profiles are plotted graphically using MATLAB for various value of Casson parameter, Magnetic parameter, Suction parameter. It is observed that the velocity is found to be decreasing with increase in Casson parameter as shown in figure.1 in presence of suction/ blowing.

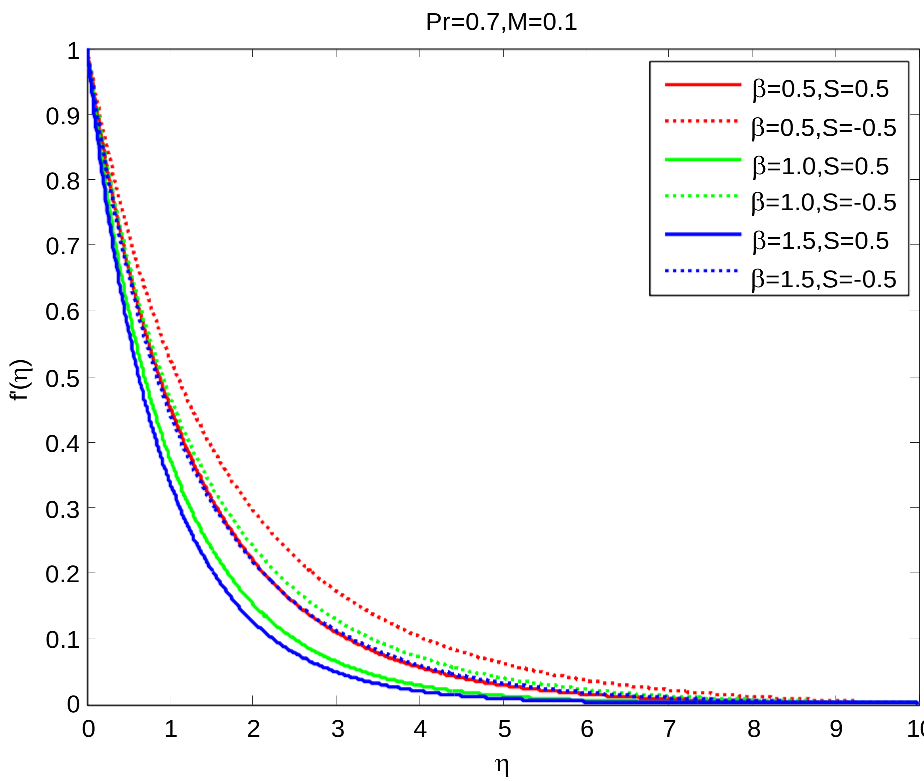

Fig.1 Velocity profiles with Casson Parameter β in presence of suction/blowing

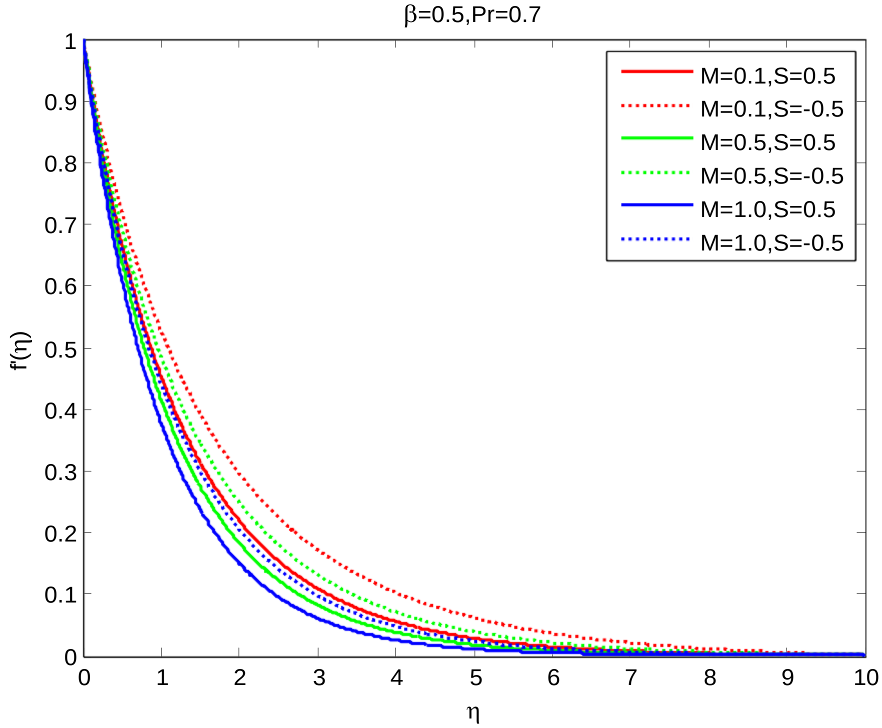

Fig.2 **Velocity profiles with Magnetic Parameter in presence of suction/blowing**

The velocity is found to be decreasing with increase in Magnetic parameter as shown in figure2 in presence of suction / blowing.

Conclusions

It is observed that

1. Velocity is found to be decreasing with increase in casson parameter in presence of suction/blowing.
2. Velocity is found to be decreasing with increase in Magnetic parameter in presence of suction/blowing.

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